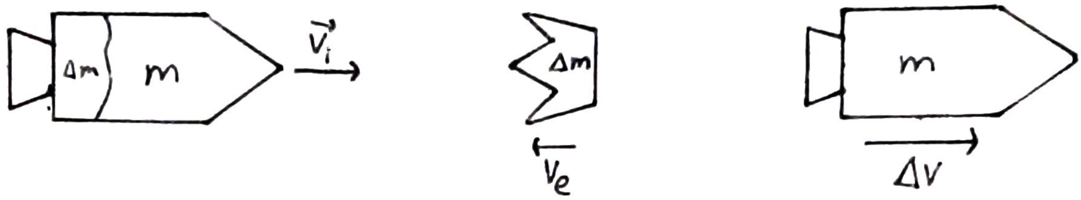
Rocket Equation

Before: After:



M = Mass of rocket
Δm = Mass of fuel ejected
Ve = Exhaust velocity

$\Delta P = 0 \quad P_i = P_f$

$$(m + \Delta m)V_i = m(V_i + \Delta V) + \Delta m(V_i - V_e)$$

$$mV_i + \Delta m V_i = mV_i + m\Delta V + \Delta m V_i - \Delta m V_e$$

$$0 = m\Delta V - \Delta m V_e \Rightarrow m\Delta V = \Delta m V_e$$

$$\lim_{\substack{\Delta m \to 0 \\ \text{and} \\ \Delta V \to 0}} (m\Delta V = \Delta m V_e) = m\,dv = dm\,V_e \Rightarrow dv = \frac{V_e}{m}\,dm$$

$$\int_{V_i}^{V_f} dv = \int_{m_i}^{m_f} \frac{V_e}{m}\,dm = V_f - V_i = V_e \int_{m_i}^{m_f} \frac{1}{m}\,dm = \Delta V = V_e\left[\ln m \Big|_{m_i}^{m_f}\right]$$

$$\Delta V = V_e(\ln m_f - \ln m_i) = V_e \ln\left(\frac{m_f}{m_i}\right)$$

Since $m_i > m_f$, ln will always produce a negative, but we know ΔV will always be positive so we can multiply it by a negative which is the same as inversing $\frac{m_f}{m_i}$

$$\boxed{\Delta V = V_e \ln\left(\frac{m_i}{m_f}\right)}$$