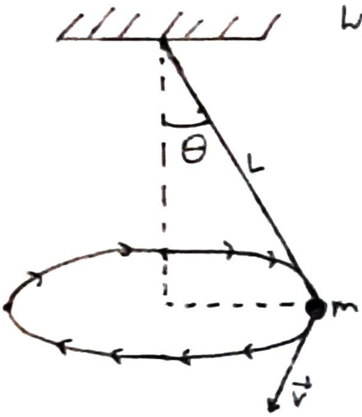
Circular Forces

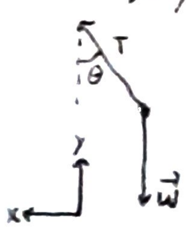
There are no frictional/centripital forces.
The centripital acceleration can be explained by all the forces in the system.

Ex: Conical Pendulum



What is θ?

Free body diagram:



$\vec{F}_{net} = m\vec{a}$

$$T\sin\theta\,\hat{x} + T\cos\theta\,\hat{y} + W(-\hat{y}) = ma_c\,\hat{x} + 0\hat{y}$$

$\hat{x}: T\sin\theta = ma_c$
$\hat{y}: T\cos\theta - W = 0 \Rightarrow T\cos\theta = W$

Solve for T, then plug into other formula to get θ. You could also divide both equations to cancel out the T.

$$\frac{T\sin\theta}{T\cos\theta} = \frac{ma_c}{mg} = \frac{v^2}{rg}$$

$r = L\sin\theta$

$$\frac{\sin\theta}{\cos\theta} = \frac{v^2}{gL\sin\theta} \Rightarrow \frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{gL}$$

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v^2}{gL} \Rightarrow 0 = \cos^2\theta + \frac{v^2}{gL}\cos\theta - 1$$

$a = 1 \quad b = \frac{v^2}{gL} \quad c = -1$

$$\cos\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\theta = \cos^{-1}\left(\frac{-\frac{v^2}{gL} \pm \sqrt{\left(\frac{v^2}{gL}\right)^2 + 4}}{2}\right)$$