Angular Momentum

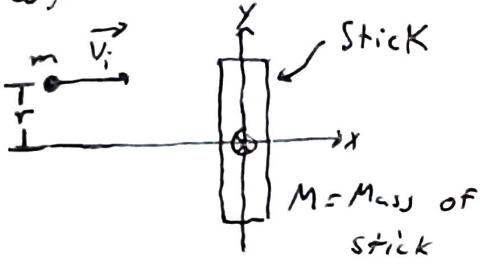
$\vec{\tau} = \vec{r} \times \vec{F}$ $\quad \vec{F} = \frac{d\vec{p}}{dt}$

$\vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$ $\quad \boxed{\vec{L} = \vec{r} \times \vec{p}}$

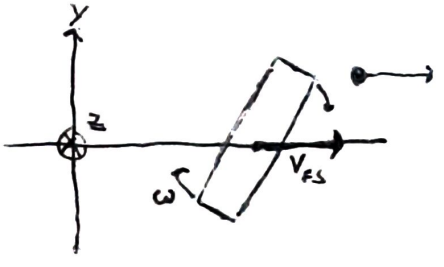
$$\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$$

Ex: What is the final velocity of the particle (V_f), the final velocity and final angular velocity of the stick (V_{fs}, ω)?

Before:



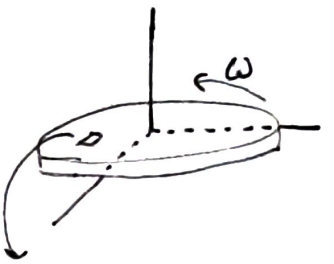
After:



3 equations, 3 unknowns

$$V_f = \frac{I(m-M) + mMr^2}{I(m+M) + mMr^2} V_i$$

$$V_{fs} = \frac{2Im}{I(m+M) + mMr^2} V_i$$



$d\vec{L} = \vec{r} \times d\vec{p}$

$dL = r(dm\, v)$ $\quad v = r\omega$

$dL = r\, dm\, r\omega$

$dL = \omega r^2 dm$

$$L = \int \omega r^2 dm = \omega \int r^2 dm$$

$$\boxed{L = I\omega}$$

$\Delta P = 0$ $\quad P_i = P_f$

$m\vec{V_i} = m\vec{V_f} + M\vec{V_{fs}}$

$\Delta L = 0$ $\quad L_i = L_f$

$\vec{r} \times \vec{p_i} = \vec{r} \times \vec{p_f} + I\vec{\omega}$

$r m V_i \hat{z} = r m V_f \hat{z} + I\omega\hat{z}$

$\Delta K = 0$ $\quad K_i = K_f$

$\frac{1}{2} m V_i^2 = \frac{1}{2} m V_f^2 + \frac{1}{2} M V_{fs}^2 + \frac{1}{2} I\omega^2$

$$\omega = \frac{2mMr}{I(m+M) + mMr^2} V_i$$