Work/Energy

When Force is constant: $\boxed{W = \vec{F} \cdot \vec{\Delta x}} = F \Delta x \cos\theta$

When Force isn't constant: $\boxed{W = \int \vec{F} \cdot dx}$

Assuming same direction.

chain rule

$$\frac{dv}{dt} = \frac{dv}{dx}\,\frac{dx}{dt}$$

$$W = \int \vec{F} \cdot dx = \int F_x\, dx = \int ma\, dx = m \int \frac{dv}{dt}\, dx$$

$$= m \int \frac{dv}{dx}\,\frac{dx}{dt}\, dx = m \int v \frac{dv}{dx}\, dx = m \int_{\text{initial}}^{\text{final}} v\, dv = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta K$$

Kinetic energy

$\boxed{K = \frac{1}{2} m v^2}$

$$W_{gravity} = \int_i^f -mg\, dx = -mg \int_i^f dx = -(mgx_f - mgx_i) = -\Delta U$$

Potential energy

$\boxed{U_{gravity} = mgh}$

$\boxed{U_{spring} = \frac{1}{2} k x^2}$ $\quad W_{spring} = \int kx\, dx = \frac{1}{2} k x^2$

A force is <u>conservative</u> if its work depends only on its starting and endpoints, allowing it to be defined with potential energy.

Ex. Gravity, springs

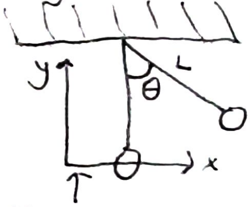
Ex of non-conservative: Friction, air resistance, force applied.

$\Delta K = W_c + W_{nc}$ $\qquad W_c = -\Delta U$ $\qquad \Delta K = -\Delta U + W_{nc}$

$\Delta K + \Delta U = W_{nc}$

$\boxed{\Delta E = \emptyset \text{ for only conservative forces}}$ $\qquad \boxed{\Delta E = W_{nc} \text{ for non-conservative forces}}$

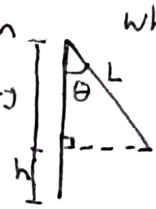
Using these is often easier than using force. Ex:



$U = \emptyset$ @ $y = \emptyset$

m is given

$V_i = \emptyset$



what's V_f?

$\cos\theta = \frac{L_y}{L}$ $\qquad h = L - L_y = L - L\cos\theta$

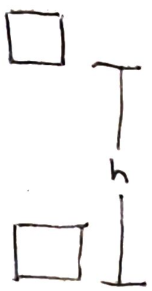
$\Delta E = \emptyset$ $\qquad E_i = E_f$ $\qquad U_i + K_i = U_f + K_f$

$mgh + \frac{1}{2} m \emptyset^2 = mg\emptyset + \frac{1}{2} m v_f^2$

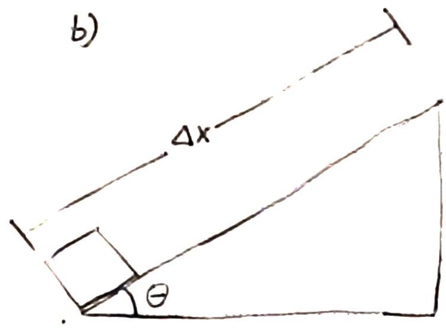
$\cancel{m}gh = \frac{1}{2}\cancel{m} v_f^2$ $\qquad v_f = \sqrt{2gh}$

Ex: Which paths do less work?

a)

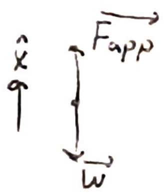


b)



$a = 0$ so you don't do extra work

a)



$\vec{F_{net}} = m\vec{a}$

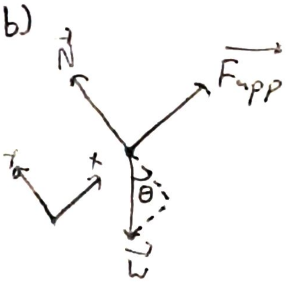
$F_{app}\hat{x} - W\hat{x} = 0$

$F_{app} = W = mg$

$W_{app} = \vec{F_{app}} \cdot \vec{\Delta x} = F_{app} h$

$W_{app} = mgh$

b)

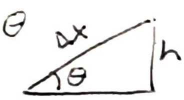


$\vec{F_{net}} = m\vec{a}$

$F_{app}\hat{x} - W\sin\theta\,\hat{x} + N\hat{y} - W\cos\theta\,\hat{y} = 0$

$\hat{x}: F_{app} - W\sin\theta = 0$

$F_{app} = W\sin\theta = mg\sin\theta$

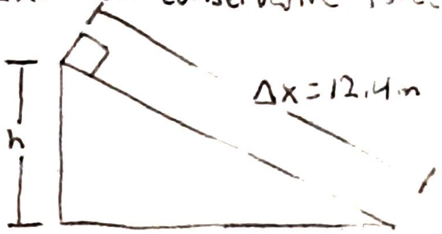


$h = \sin\theta\,\Delta x$

$W_{app} = F_{app}\Delta x = mg(\sin\theta\,\Delta x) = mgh$

Ramps use the same work, but since it's over a longer distance, the force applied is lower.

Ex: Non-conservative forces



$h = 2.6m$

$V_i = 1.4 \frac{m}{s}$ $m = 47 kg$

$V_f = 6.2 \frac{m}{s}$ f = Friction/air resistance $= 41N$

what is the F_{app}?

$$\Delta E = W_{nc}$$

$$E_f - E_i = W_{nc}$$

$$(U_f + K_f) - (U_i + K_i) = W_{nc}$$

$$(mg0 + \frac{1}{2}mV_f^2) - (mgh + \frac{1}{2}mV_i^2) = W_{nc}$$

$W_{nc} = (-f + F_{app})\Delta x$ $\frac{1}{2}mV_f^2 - mgh - \frac{1}{2}mV_i^2 = -\Delta x f + \Delta x F_{app}$

$$F_{app} = \frac{\frac{1}{2}mV_f^2 - mgh - \frac{1}{2}mV_i^2 + \Delta x f}{\Delta x} = 13.558 N$$

$W_{app} = F_{app}\Delta x = 168.12J$

$\text{Power} = \frac{dE}{dt}$ $W = \vec{F} \cdot \vec{\Delta x} = -\Delta U$

$F\,dx = -dU$

$F = -\frac{dU}{dx}$

For kenetic energy:

$v^2 = \vec{v} \cdot \vec{v} = V_x^2 + V_y^2$