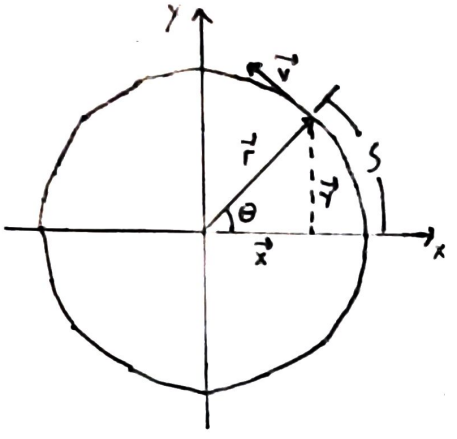
Uniform Circular Motion

An object moves around a circle at constant speed (v).



S = Arc length = $2\pi r \cdot \frac{\theta\,deg}{360} = 2\pi r \cdot \frac{\theta\,rad}{2\pi} = r\theta$ Circumference = $2\pi r$

$$\boxed{S = r\theta}$$

$$V = \frac{S}{t} = \frac{r\theta}{t} \Rightarrow \boxed{\theta = \frac{V}{r}t = \omega t}$$

$$\vec{r} = \vec{x} + \vec{y} = x\hat{x} + y\hat{y} \qquad \vec{r} = r\cos(\omega t)\hat{x} + r\sin(\omega t)\hat{y}$$

$$x = r\cos(\theta) = r\cos(\omega t)$$
$$y = r\sin(\theta) = r\sin(\omega t)$$

$$\vec{V} = \frac{d\vec{r}}{dt} = -r\omega\sin(\omega t)\hat{x} + r\omega\cos(\omega t)\hat{y}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \underbrace{-r\omega^2\cos(\omega t)\hat{x}}_{a_x} \underbrace{- r\omega^2\sin(\omega t)\hat{y}}_{a_y}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(-r\omega^2\cos(\omega t)\right)^2 + \left(-r\omega^2\sin(\omega t)\right)^2}$$

$$= \sqrt{r^2\omega^4\cos^2(\omega t) + r^2\omega^4\sin^2(\omega t)} = r\omega^2\sqrt{\cos^2(\omega t) + \sin^2(\omega t)}$$

$$\cos^2\theta + \sin^2\theta = 1 \Rightarrow \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

$$\boxed{a = r\omega^2} \qquad V = r\omega \qquad \omega = \frac{V}{r} \qquad a = r\frac{V^2}{r^2} \qquad \boxed{a = \frac{V^2}{r}}$$

T = Period. Time to make a full rotation.

$$2\pi r = VT \Rightarrow \boxed{T = \frac{2\pi r}{V} = \frac{2\pi}{\omega}}$$