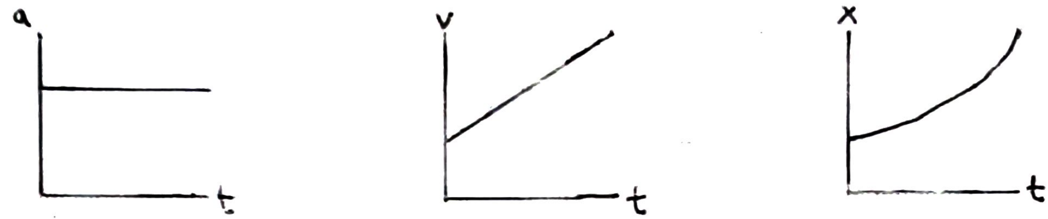
Kinematic equations

When a is constant.



$$V_f = \int a \, dt \qquad X_f = \int at + V_i \, dt$$

$$t = \frac{V_f - V_i}{a} \longleftarrow \boxed{V_f = at + V_i} \qquad \boxed{X_f = \tfrac{1}{2}at^2 + V_i t + X_i}$$

$$X_f = \frac{1}{2}a\left(\frac{V_f - V_i}{a}\right)^2 + V_i\left(\frac{V_f - V_i}{a}\right) + X_i$$

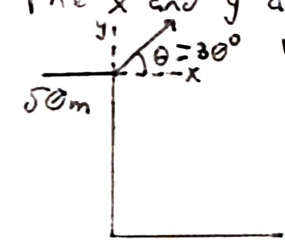
$$X_f - X_i = \frac{1}{2}a\frac{(V_f - V_i)^2}{a^2} + \frac{V_i(V_f - V_i)}{a} = \frac{(V_f - V_i)^2}{2a} + \frac{2V_i(V_f - V_i)}{2a}$$

$$= \frac{(V_f - V_i)^2 + 2V_i(V_f - V_i)}{2a} \qquad (V_f - V_i)(V_f - V_i) = V_f^2 - 2V_iV_f + V_i^2$$
$$(2V_i)(V_f - V_i) = 2V_iV_f - 2V_i^2$$

$$\Delta X = \frac{V_f^2 - 2V_iV_f + V_i^2 + 2V_iV_f - 2V_i^2}{2a} = \frac{V_f^2 - V_i^2}{2a} \qquad \boxed{V_f^2 = V_i^2 + 2a\Delta X}$$

2D motion

The x and y acceleration are separate. $a_x = 0$ and $a_y = -g = -9.8 \frac{m}{s^2}$



$V_i = 10 \frac{m}{s}$

a) How far above the roof does it go?

$V_{yi} = V_i \sin\theta \quad V_{yf} = 0 \quad V_{yf}^2 = V_{yi}^2 + 2a\Delta y$

$0^2 = (V_i \sin\theta)^2 + 2(-g)\Delta y \quad \Delta y = 1.28\,m$

b) How long does it take to reach the ground?

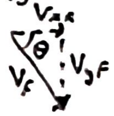
$Y_f = \frac{1}{2}at^2 + V_{yi}t + Y_i \qquad -50 = \frac{1}{2}(-9.8)t^2 + V_i\sin\theta t + 0 \qquad 0 = -4.9t^2 + 5t + 50$

$a = -4.9 \quad b = 5 \quad c = 50$

$$t = \frac{-5 \pm \sqrt{5^2 - 4(-4.9)(50)}}{2(-4.9)}$$

$t = \cancel{-2.72}$ and $3.75\,s$

c) What angle does it hit the ground?



$V_{xf} = V_{xi} = V_i \cos\theta \qquad V_{yf} = at + V_{yi}$

$\tan\theta = \frac{V_{yf}}{V_{xf}} \qquad \theta = -74.74^\circ$

d) How far away from the building does it hit?

$X_f = \frac{1}{2}at^2 + V_{xi}t + X_i \qquad X_f = 0 + V_i\cos\theta t + 0 \qquad X_f = 32.475\,m$