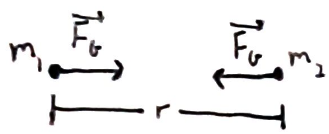
Gravity

Force of gravity



$$\boxed{F_G = \frac{G m_1 m_2}{r^2}}$$

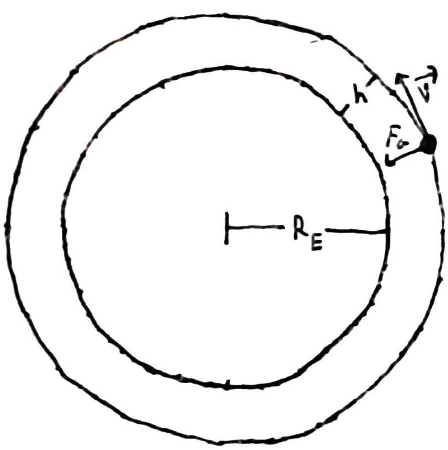
$$\boxed{G = 6.674E-11}$$

Force of gravity near earth's surface

$$F_G = mg = \frac{GM_E m}{R_E^2} \Rightarrow g = \frac{GM_E}{R_E^2}$$

M_E = Mass of earth
R_E = Radius of earth

What height (h) for geostationary orbit around earth? → Satellite stays in the same position over earth.



$$R = R_E + h$$

$$\vec{F_{net}} = m\vec{a}$$

$$F_G = ma_c$$

$$\frac{GM_E m}{R^2} = m\frac{v^2}{R} \Rightarrow v = \sqrt{\frac{GM_E}{R}} = \sqrt{\frac{GM_E}{R_E + h}}$$

$$T = 1\text{day} \cdot \frac{24\text{h}}{1\text{day}} \cdot \frac{60\text{min}}{1\text{h}} \cdot \frac{60\text{sec}}{1\text{min}} = 86{,}400\text{sec}$$

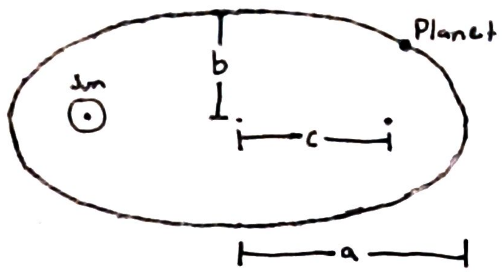
$$T = \frac{2\pi R}{v} \Rightarrow v = \frac{2\pi(R_E + h)}{T} = \sqrt{\frac{GM_E}{R_E + h}} \Rightarrow \frac{4\pi^2(R_E + h)^2}{T^2} = \frac{GM_E}{R_E + h}$$

$$\Rightarrow \frac{4\pi^2(R_E + h)^3}{T^2} = GM_E \Rightarrow (R_E + h)^3 = \frac{GM_E T^2}{4\pi^2} \Rightarrow h = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}} - R_E$$

$$h = 35{,}865 \text{ km}$$

Kepler's laws of planetary motion

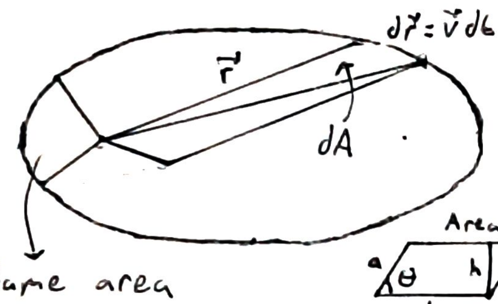
Law 1: Planets move in elliptical orbits with the sun at one focus



b = Semi-minor axis

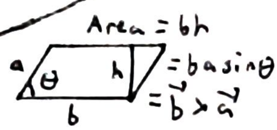
a = Semi-major axis

Law 2: A line between the sun and the planet sweeps out equal areas of its elliptical orbit in equal intervals of time.



L is constant

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times M\vec{v} = M(\vec{r} \times \vec{v})$$



$$\frac{L}{M} = \vec{r} \times \vec{v}$$

$$dA = \frac{1}{2}(\vec{r} \times d\vec{r}) = \frac{1}{2}(\vec{r} \times \vec{v}\,dt) = \frac{1}{2}(\vec{r} \times \vec{v})\,dt = \frac{1}{2}\left(\frac{L}{M}\right)dt \qquad \boxed{\frac{dA}{dt} = \frac{L}{2M}}$$

Law 3: The square of the orbital period is proportional to the semi-major axis cubed.

Assume circular orbit: $F_G = m a_c = m\frac{v^2}{r} \Rightarrow \frac{GMm}{r^2} = \frac{m}{r}\frac{4\pi^2 r^2}{T^2}$

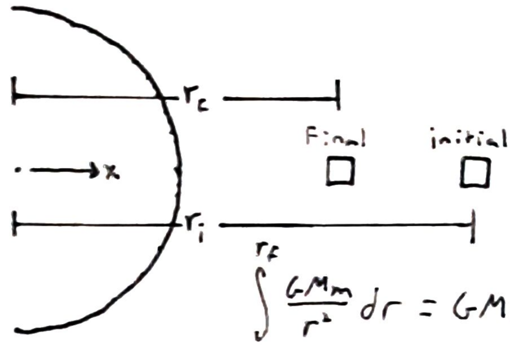
$T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T}$

$$\Rightarrow \frac{GM}{r^2} = \frac{4\pi^2 r}{T^2} \Rightarrow \frac{GM}{r^3} = \frac{4\pi^2}{T^2}$$

$$\boxed{T^2 = \frac{4\pi^2}{GM} r^3}$$

The semi-major axis when it's an ellipse.

Gravitational potential energy



$$\Delta U = -W_{external} = -\int_{r_i}^{r_f} F_G \cdot dr$$

$$F_G = -\frac{GMm}{r^2}$$

$$\int_{r_i}^{r_f} \frac{GMm}{r^2}\,dr = GMm\int_{r_i}^{r_f} \frac{1}{r^2}\,dr = GMm\left[-\frac{1}{r}\Big|_{r_i}^{r_f}\right] = GMm\left(-\frac{1}{r_f} + \frac{1}{r_i}\right)$$

$$\Delta U = U_f - U_i = \left(-\frac{GMm}{r_f}\right) - \left(-\frac{GMm}{r_i}\right) \Rightarrow \boxed{U = -\frac{GMm}{r}}$$

Escape velocity:

$$\Delta E = 0 \quad E_i = E_f \quad U_i + k_i = U_f + k_f$$

$$-\frac{GMm}{r} + \frac{1}{2}m V_{esc}^2 = -\frac{GMm}{\infty} + \frac{1}{2}m\,0^2$$

$$-\frac{2GMm}{r} + mV_{esc}^2 = 0 \Rightarrow -\frac{2GM}{r} + V_{esc}^2 = 0$$

$$\boxed{V_{esc} = \sqrt{\frac{2GM}{r}}}$$

For earth: $V_{esc} = 11{,}185\ \frac{m}{s}$

At this velocity, a satellite won't stay in orbit around the planet.