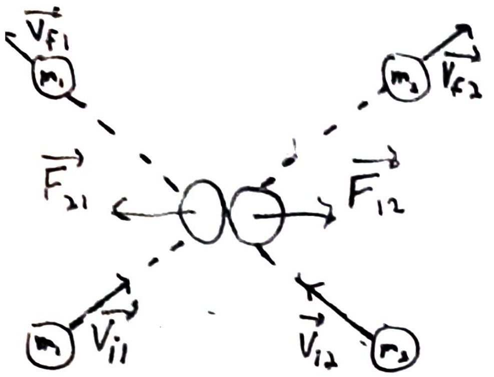
Collisions



$$\vec{F}_{21} = -\vec{F}_{12} \quad \text{Newton's 3rd Law}$$
$$\vec{F}_{21} + \vec{F}_{12} = 0$$
$$m_1\vec{a}_1 + m_2\vec{a}_2 = 0 \quad \text{Newton's 2nd Law}$$
$$m_1\frac{d\vec{V}_1}{dt} + m_2\frac{d\vec{V}_2}{dt} = 0$$
$$\frac{d}{dt}\underbrace{\left(m_1\vec{V}_1 + m_2\vec{V}_2\right)} = 0 \qquad \boxed{\vec{P} = m\vec{V}} \qquad \boxed{\Delta\vec{P} = 0}$$

The total momentum stays constant w/ respect to time.

$$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V}) = \frac{d\vec{P}}{dt}$$

when mass is constant

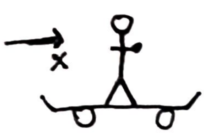
$$\boxed{\vec{F}_{net} = \frac{d\vec{P}}{dt}}$$

Ex: You throw a ball on a frictionless skateboard.

$V_i = 0 \quad V_b = 10\,\frac{m}{s}$

m_b = Mass of ball = 3kg. After you throw, what v do you move back?

m_p = Mass of person = 50 kg

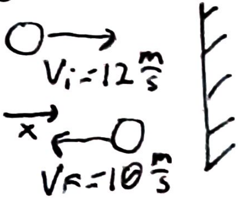


$\Delta P = 0$

$P_f = P_i$

$$m_p V_p + m_b V_b = (m_p + m_b)V_i$$
$$V_p = \frac{(m_p + m_b)V_i - m_b V_b}{m_p} = \frac{0 - (3)(10)}{50} = -\frac{3}{5}\,\frac{m}{s}$$

Ex: A ball is throw at a wall. Suppose it was in contact for 0.0001 s. What average force gave it that acceleration?



$m = 0.2\,kg$

$$\vec{F}_{avg} = \frac{\Delta P}{\Delta t} = \frac{P_f - P_i}{t} = \frac{mV_f - mV_i}{t} = 0.2\,\frac{-10-12}{0.0001}$$
$$= -44{,}000\,N$$

Elastic - Objects bounce off. $\Delta P = 0$ $\Delta K = 0$

Inelastic - Objects bounce off, but some kenetic energy is lost.

$\Delta P = 0$ $\Delta K \neq 0$

Perfectly inelastic - Objects stick together after collision.

Before:

$O \rightarrow$ $\leftarrow O$

m_1 m_2

$\vec{V}_{i1}$ $\vec{V}_{i2}$

After:

Elastic: $\vec{V}_{f1} \leftarrow O$ $O \rightarrow \vec{V}_{f2}$

$$\Delta P = 0$$

$$P_i = P_f$$

$$m_1 \vec{V}_{i1} + m_2 \vec{V}_{i2} = m_1 \vec{V}_{f1} + m_2 \vec{V}_{f2}$$

$$m_1 \vec{V}_{i1} - m_1 \vec{V}_{f1} = m_2 \vec{V}_{f2} - m_2 \vec{V}_{i2}$$

$$(1) \quad m_1 (\vec{V}_{i1} - \vec{V}_{f1}) = m_2 (\vec{V}_{f2} - \vec{V}_{i2})$$

$$\Delta K = 0$$

$$K_i = K_f$$

$$\tfrac{1}{2} m_1 \vec{V}_{i1}^{\,2} + \tfrac{1}{2} m_2 \vec{V}_{i2}^{\,2} = \tfrac{1}{2} m_1 \vec{V}_{f1}^{\,2} + \tfrac{1}{2} m_2 \vec{V}_{f2}^{\,2}$$

$$m_1 \vec{V}_{i1}^{\,2} + m_2 \vec{V}_{i2}^{\,2} = m_1 \vec{V}_{f1}^{\,2} + m_2 \vec{V}_{f2}^{\,2}$$

$$m_1 \vec{V}_{i1}^{\,2} - m_1 \vec{V}_{f1}^{\,2} = m_2 \vec{V}_{f2}^{\,2} - m_2 \vec{V}_{i2}^{\,2}$$

$$m_1 (\vec{V}_{i1}^{\,2} - \vec{V}_{f1}^{\,2}) = m_2 (\vec{V}_{f2}^{\,2} - \vec{V}_{i2}^{\,2})$$

$$m_1 (\vec{V}_{i1} - \vec{V}_{f1})(\vec{V}_{i1} + \vec{V}_{f1}) = m_2 (\vec{V}_{f2} - \vec{V}_{i2})(\vec{V}_{f2} + \vec{V}_{i2})$$

$$(2) \quad \vec{V}_{i1} + \vec{V}_{f1} = \vec{V}_{f2} + \vec{V}_{i2}$$

Use equations (1) and (2) to solve for:

$$\vec{V}_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \vec{V}_{i1} + \left(\frac{2m_2}{m_1 + m_2}\right) \vec{V}_{i2}$$

$$\vec{V}_{f2} = \left(\frac{2m_1}{m_1 + m_2}\right) \vec{V}_{i2} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) \vec{V}_{i2}$$