Maxwell's 1st law - Guass's law for $\vec{E}$ - $\oint \vec{E} \cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$

Maxwell's 2nd law - Guass's law for $\vec{B}$ - $\oint \vec{B} \cdot d\vec{A} = \emptyset$

- No matter the closed surface, there's never a point src of magnetic fields.
- Magnetic fields always form closed loops.

Biot-Savart law (B.S. Law)

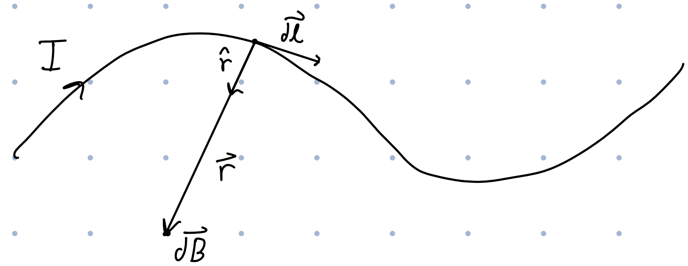
$\vec{E} = \int d\vec{E}$ (POS)

$= \int \frac{k\,dq}{r^2}\hat{r}$

$\vec{B} = \int d\vec{B}$ (POS)

$= \int \boxed{\frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}}$

Ex:



$d\hat{B} = d\hat{l} \times \hat{r} = \hat{\otimes}$

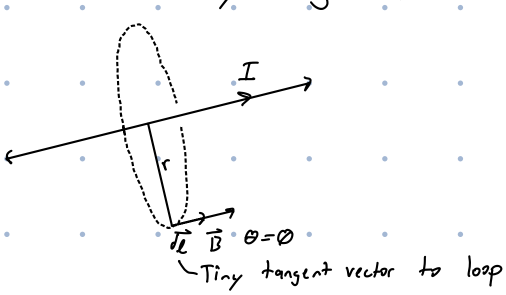
Ampere's law incomplete: (Maxwell's 3rd equation)

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{net}}$$

$I_{net} = I$ going in/out of a plane (surface enclosed by an Amperic loop)

- Currents create magnetic fields.

Ex: Infintly long line of I



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

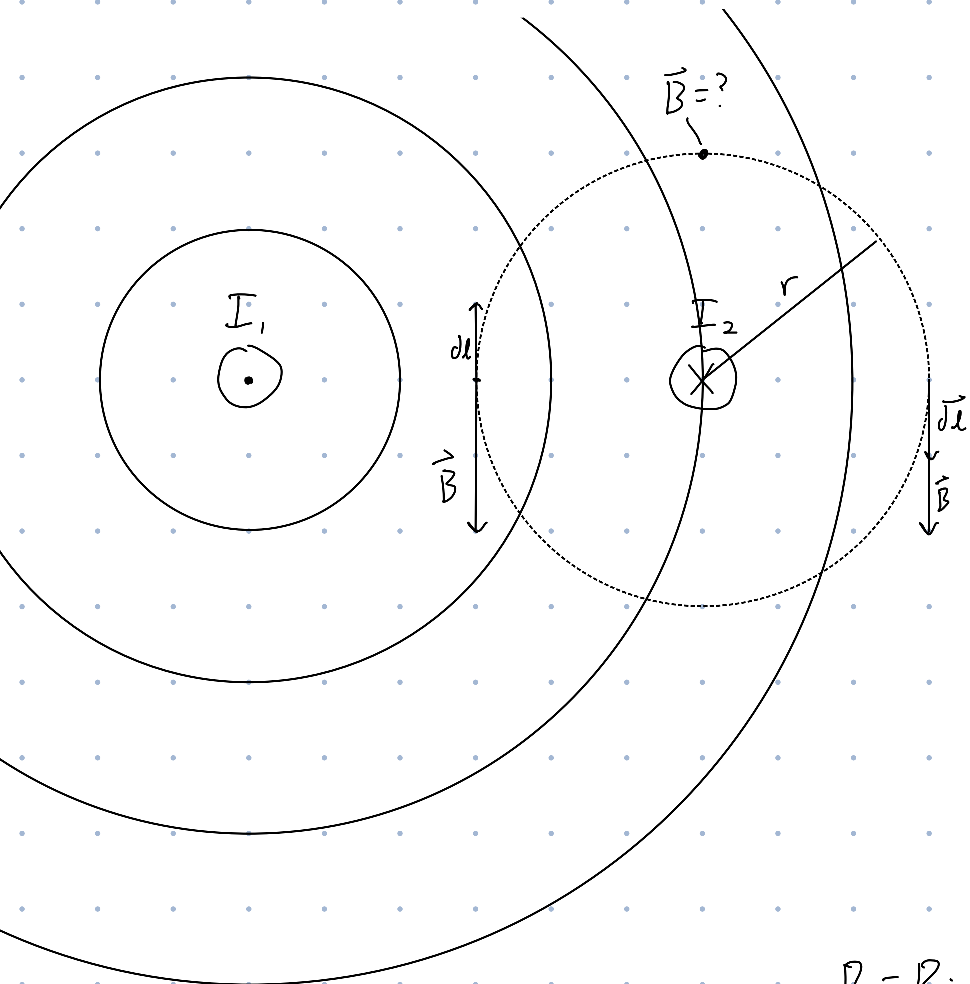
$\oint B\,dl\cos\emptyset = \mu_0 I$

$B \underbrace{\oint dl}_{\text{Arc length of chosen loop}} = \mu_0 I$

$B 2\pi r = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi r}$

Ex: 2 infinitly long wires



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{net}$$

$\downarrow$ Pos

$$\oint \left(\vec{B_{in}} + \vec{B_{out}}\right) \cdot \vec{dl} = \mu_0 I_{net}$$

$$\oint \vec{B_{in}} \cdot \vec{dl} + \underbrace{\oint \cancel{\vec{B_{out}}} \cdot \vec{dl}}_{0\ (I_{net\ out} = 0)} = \mu_0 I_{net}$$

3 step checklist

- $\vec{B_{out}} = 0$ (Not true)
- $\vec{B_{out}} \perp \vec{dl}$ (Not true)
- ✓ All $\vec{B_{out}} \cdot \vec{dl}$ s added up cancel out and equal to 0. (Remember the cube)

$$B = B_{in}$$

$$\oint B \, dl \cos \cancel{\theta}^{\,1} = \mu_0 I_2$$

Constant at a given r

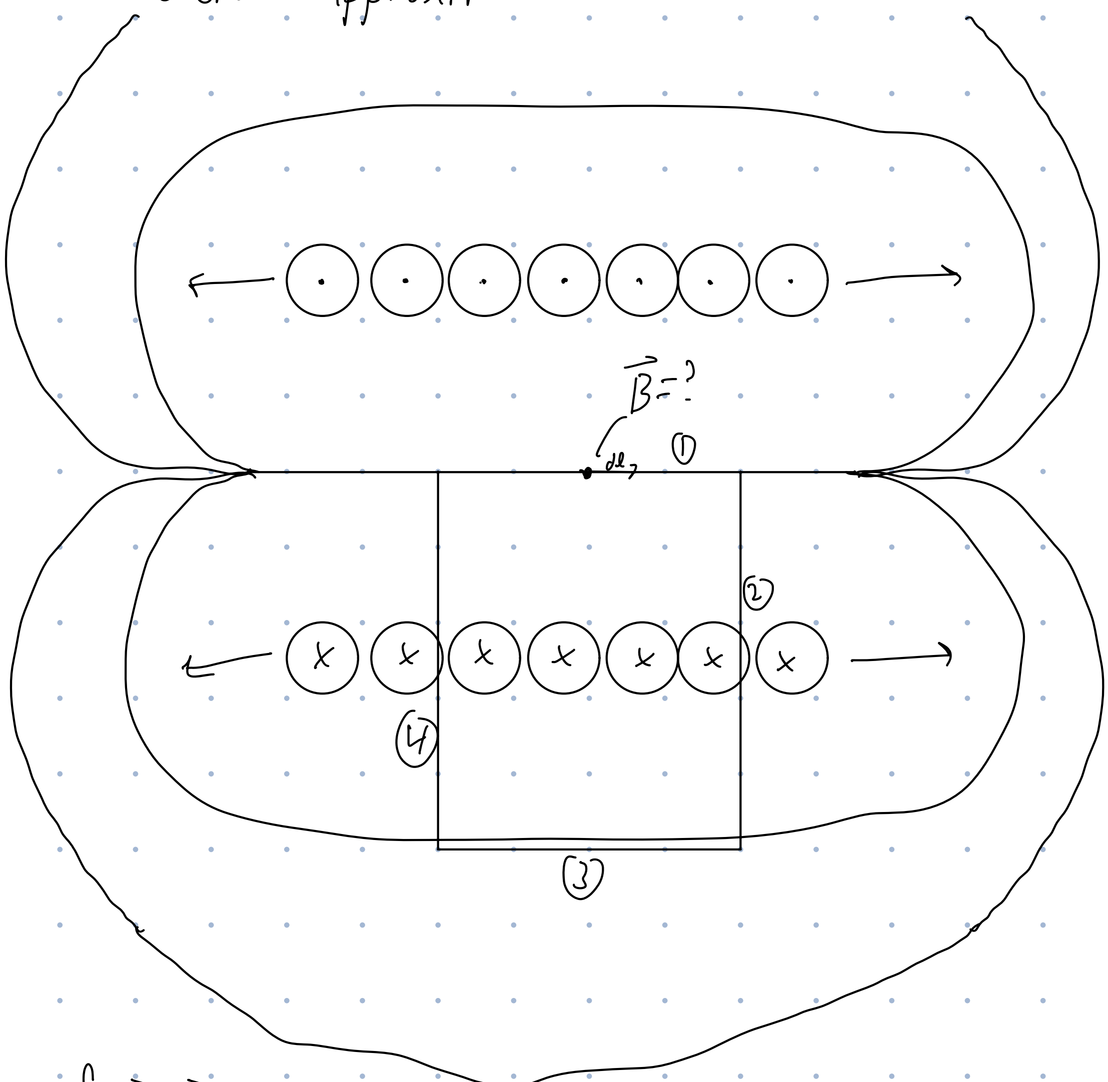
$$B \oint dl = \mu_0 I_2$$

$$B \, 2\pi r = \mu_0 I_2$$

$$B = \frac{\mu_0 I_2}{2\pi r}$$

Solenoids

- A solenoid is a wire wrapped around a shaft where the length is long compared to its diameter.
- Solenoid approximation - Inside uniform and outside $\vec{B} = 0$.



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{net}$$

Solenoid approximation

$$\int_1 \vec{B}\cdot d\vec{l} + \overset{0\,(\vec{B}\perp d\vec{l})}{\cancel{\int_2 \vec{B}\cdot d\vec{l}}} + \overset{0\,(\vec{B}=0)}{\cancel{\int_3 \vec{B}\cdot d\vec{l}}} + \overset{0\,(\vec{B}\perp d\vec{l})}{\cancel{\int_4 \vec{B}\cdot d\vec{l}}}$$

$$\int B\,dl\,\cancel{\cos 0}^{\,1} = \mu_0 I_{net}$$

Uniform

$$B\int dl = \mu_0 nLI$$

$$BL = \mu_0 nLI \rightarrow B = \mu_0 nI$$

Changes if you wrap around different matterials.

$$n = \frac{\#\text{ of turns}}{\text{length}} = \frac{N}{L}$$

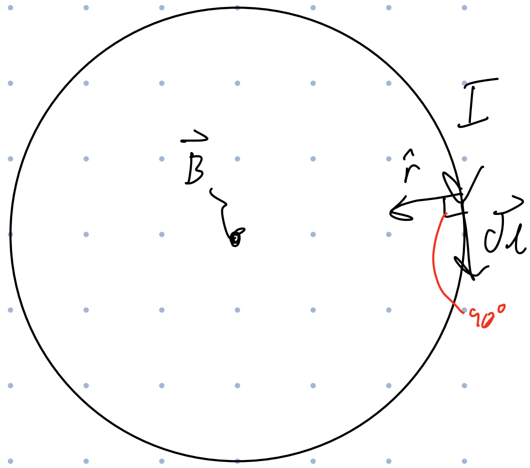
$I_{net} = NI$ Current from 1 wire

# of turns in amperic loop

$= nLI$

What about a circle of charge spinning? — What about here?

P



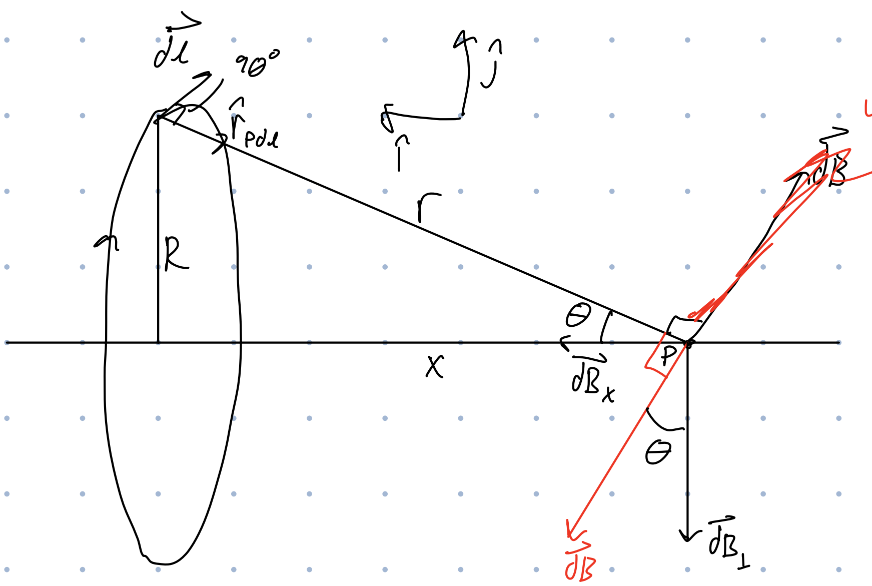
$$\vec{B} = \int \vec{dB} \quad (POS)$$

$$= \int \frac{\mu_0}{4\pi} \cdot \frac{I\vec{dl}\times\hat{r}}{r^2}$$

$$|\vec{B}| = \frac{\mu_0 I}{4\pi r^2}\int dl\,(1)\cancel{\sin 90^\circ}^{\,1}$$

$$= \frac{\mu_0 I}{\cancel{4}_{2}\cancel{\pi}\, r^{\cancel{2}}}\,\cancel{2}\cancel{\pi}\cancel{r}$$

$$\vec{B} = \frac{\mu_0 I}{2r}\,(\hat{\otimes})$$



use $\vec{dl}\times\vec{r} = \vec{B}$

r.h.r

$$\sin\theta = \frac{dB_x}{dB} \rightarrow dB_x = dB\sin\theta$$

$$\vec{B} = \int \vec{dB} \quad (POS)$$

$$= \int \vec{dB}_x + \cancel{\int \vec{dB}_\perp}^{\;0\,(\text{symmetry})}$$

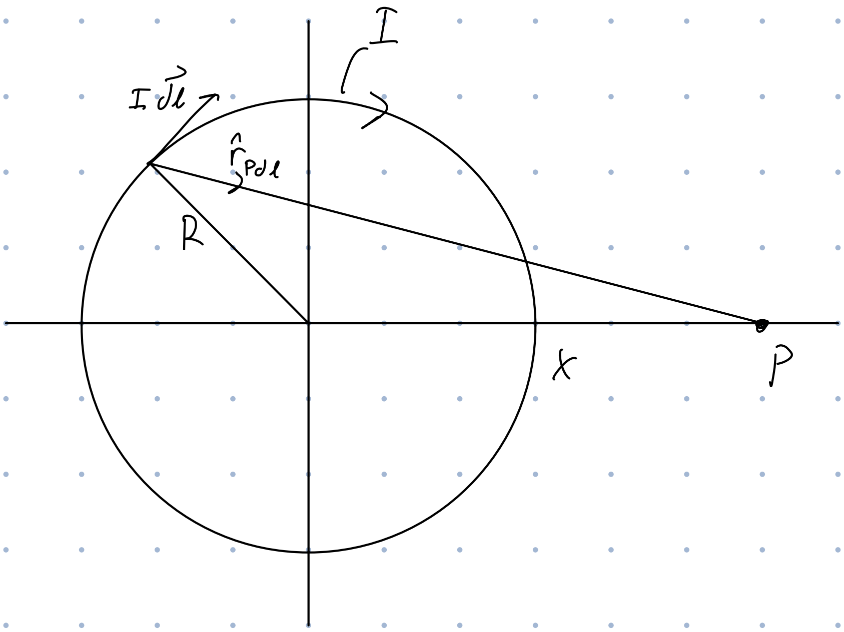
$$= \int \vec{dB}\sin\theta$$

$$= \int \frac{\mu_0}{4\pi}\,\frac{I\vec{dl}\times\hat{r}_{Pdl}}{r^2}\sin\theta$$

$$= \frac{\mu_0 I \sin\theta}{4\pi r^2} \int d\vec{l} \times \hat{r}_{Pdl}$$

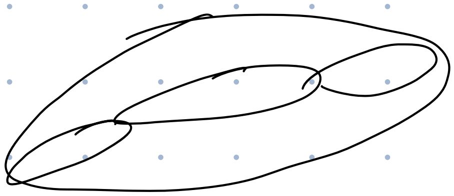
$$|\vec{B}| = \frac{\mu_0 I \sin\theta}{4\pi r^2} \int dl\,(1)\,\overset{1}{\cancel{\sin 90^\circ}}$$

$$= \frac{\mu_0 I \sin\theta}{4\pi r^2}\, 2\pi R$$



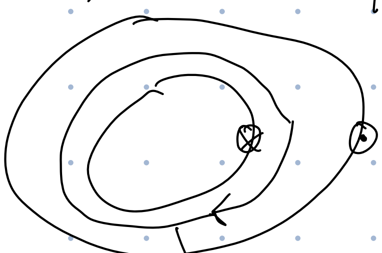
Rectangular toroid

Solonoid wrapped on itself



Top view

No magnetic fields outside the toroid.



Magnetic field not uniform.

Can use amperer's law

How?

r_o Rectangular toroid