Magnetic force

$$\boxed{\vec{F}_B = q(\vec{V} \times \vec{B})}$$

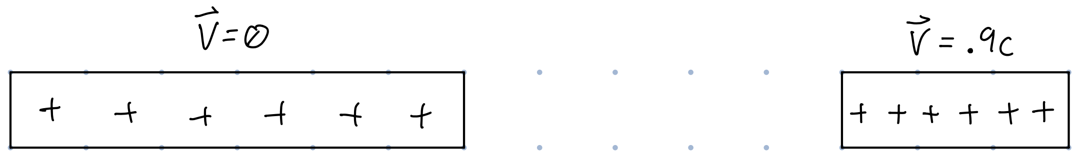
- Magnetic field (B) in units of Tesla.
- $\vec{V}$ - Velocity of q from a reference frame.

- If you're moving at the same speed as the particle (q), then you see $\vec{V}=0$. Which means you don't see a magnetic force ($\vec{F}_B = 0$).
- Two observers traveling at different speeds see different E-Fields & magnetic fields, but do see the same acceleration and thus force ($\vec{F}_L$).

$$\boxed{\vec{F}_L = \vec{F}_e + \vec{F}_B}$$

Lorentz force

Ex: From a stationary perspective, as a rob moves closer to the speed of light it shrinks.



But the charge stays the same, therefore the E-field has to increase.

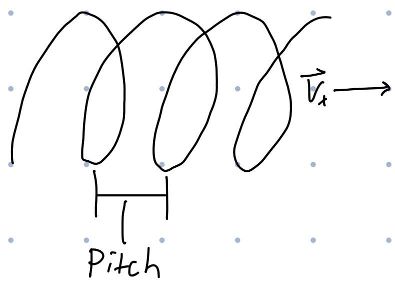
F_B Properties

1) $\vec{F}_B \perp \vec{v}$ & $\vec{F}_B \perp \vec{B}$

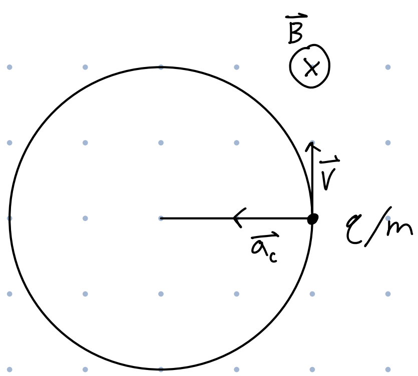
2) $\vec{F}_B$ doesn't do work.

- It doesn't change kinetic energy.
- A q in a uniform static B-field goes in a circle.

  or helix :



Ex: Find the period (T).



System: q

$\vec{F}_{net} = m\vec{a}_c$ (No tangential acceleration)

$$q(\vec{V} + \vec{B}) = \frac{mV^2}{r}$$

$$qVB\cancel{\sin 90^\circ}^{\,1} = \frac{mV^2}{r} \qquad \left(V = \frac{\Delta x}{\Delta t} = \frac{2\pi r}{T} \text{ since no tangential acceleration}\right)$$

$$qB = \frac{m}{r} \cdot \frac{2\pi r}{T}$$

$$T = \frac{2\pi m}{qB}$$

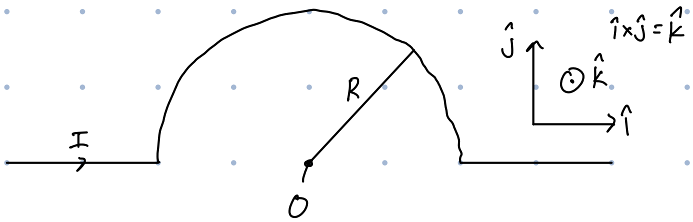
- The period doesn't depend on r or V. Meaning $\downarrow r \to \downarrow V$ and $\uparrow r \to \uparrow V$ in order to keep the T the same.

- You can use this to create a divice which accelerates charges called a cyclitron.

$$\boxed{d\vec{F}_B = \underbrace{I\,d\vec{\ell}}_{\text{Element of current}} \times \vec{B}}$$

Ex: Using component form find $\vec{F}_B$

$\otimes \vec{B}$ (Uniform and static)



$$d\vec{\ell} = dx\,\hat{\imath} + dy\,\hat{\jmath} + \overset{\emptyset \text{ (Wire on xy-plane)}}{\cancel{dz}}\,\hat{k}$$

$$\vec{B} = \emptyset\,\hat{\imath} + \emptyset\,\hat{\jmath} - B\,\hat{k}$$

$$d\vec{\ell} \times \vec{B} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ dx & dy & \emptyset \\ \emptyset & \emptyset & -B \end{bmatrix} \rightarrow \begin{matrix} -Bdy - \emptyset\,\hat{\imath} \ + \\ \emptyset + Bdx\,\hat{\jmath} \ + \\ \emptyset - \emptyset\,\hat{k} \end{matrix}$$

$$= -Bdy\,\hat{\imath} + Bdx\,\hat{\jmath} + \emptyset\,\hat{k}$$

$$\vec{F}_B = \int d\vec{F}_B$$

$$= \int I\,d\vec{\ell} \times \vec{B}$$

$$= \int I(-Bdy\,\hat{\imath} + Bdx\,\hat{\jmath})$$

I & B are uniform

$$= IB \int -dy\,\hat{\imath} + dx\,\hat{\jmath}$$

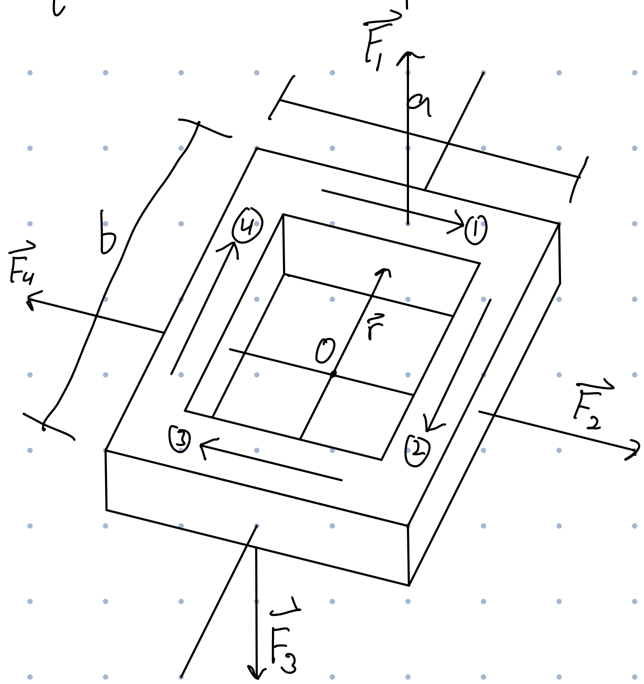
$$= IB\left[-\hat{\imath}\int_{\emptyset}^{\emptyset} dy + \int_{-R}^{R} dx\,\hat{\jmath}\right]$$

Starting $y=\emptyset$
Ending $y=\emptyset$

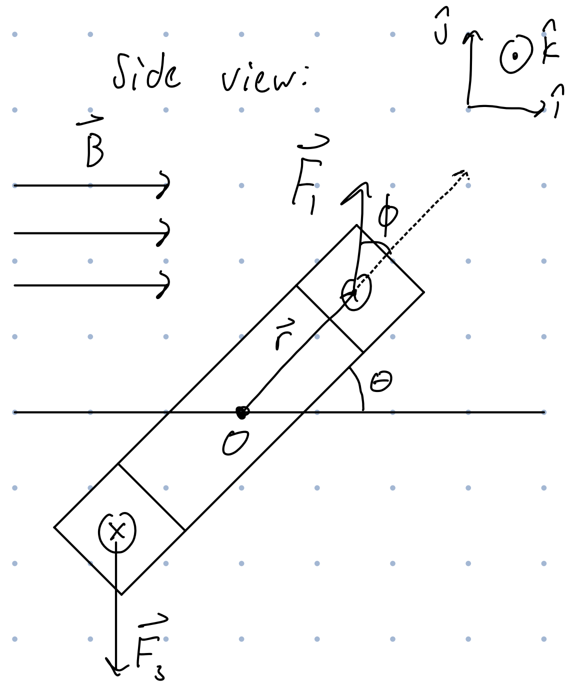
Starting $x=-R$
Ending $x=R$

$$= IB\hat{\jmath}\int_{-R}^{R} dx = 2IBR\hat{\jmath}$$

Torque on a loop of current



Side view:



System: Loop

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4$$

$$= 2\vec{\tau}_1$$

$$= \underbrace{ab}_{\text{Cross sectional area (A)}} I B \sin\phi \, \hat{k}$$

$$= \underbrace{AI}_{\text{Magnetic moment } (\mu_B)} B \sin\theta$$

$$\boxed{\mu_B = IA}$$

$\boxed{\text{Direction is r.h.r of } I}$

$$\boxed{\vec{\tau} = \vec{\mu}_B \times \vec{B}}$$

1/3: $\vec{\tau}_1 = \vec{r} \times \vec{F}_B$

$$\vec{F}_B = I\vec{l} \times \vec{B}$$

$$= IaB \cancel{\sin 90^\circ}^{\,1} \, \hat{j}$$

$$= \vec{r} \times IaB\hat{j}$$

$$= \frac{b}{2} IaB \sin\phi \, \hat{k}$$

$\vec{\tau}_1 = \vec{\tau}_3$ by symetry

2/4: $\vec{\tau}_2 = \vec{r} \times \vec{F}_B$

$$\vec{F}_B = I\vec{l} \times \vec{B}$$

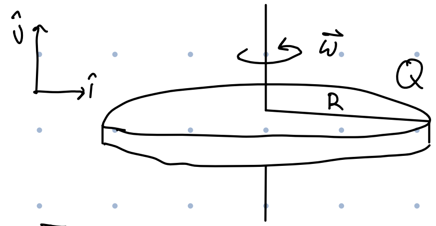
$$= IbB \sin\theta \, \hat{k}$$

$$= \frac{a}{2} IbB \sin\theta \sin 0$$

$$= 0$$

$\vec{\tau}_2 = \vec{\tau}_4$ by symetry

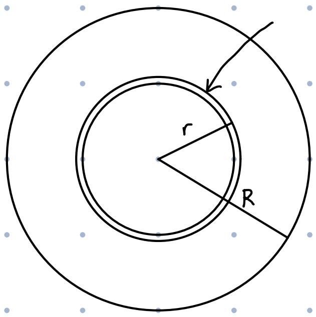
Magnetic moment of a thin spinning disk of charge.



$$\left|\vec{\mu}_B\right| = \int \left|d\vec{\mu}_B\right|$$

$$= \int dI\, A$$

Top view:



what's the little bit of current (dI) across the loop?

$$dI = \frac{\Delta Q}{\Delta t} = \frac{dQ}{T}$$

The little bit of charge in loop.

$$\sigma = \frac{Q}{A} = \frac{dQ}{2\pi r dr} \rightarrow dQ = \sigma 2\pi r dr$$

$$dI = \frac{\sigma 2\pi r dr}{T}$$

$$= \int \pi r^2 \frac{\sigma 2\pi r dr}{T}$$

$$= \frac{\sigma 2\pi^2}{T} \int_0^R r^3 dr$$

$$= \frac{\sigma 2\pi^2}{T} \left[\frac{1}{4} r^4\right]_0^R$$

$$= \frac{\sigma \pi^2}{2T} R^4 \hat{j}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$= \sigma \omega \pi R^4 \hat{j}$$

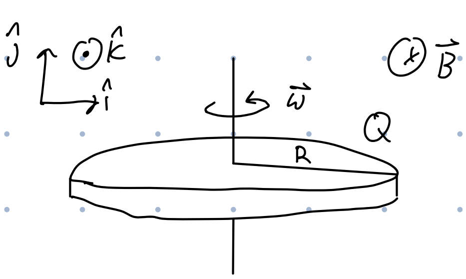
Precession

$$\vec{F} = m\vec{a} \rightarrow \vec{F} \parallel \vec{a}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \rightarrow \vec{F} \parallel \Delta\vec{p}$$

$$\vec{\tau} = I\vec{\alpha} \rightarrow \vec{\tau} \parallel \vec{\alpha}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \rightarrow \boxed{\vec{\tau} \parallel \Delta\vec{L}} \star$$

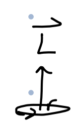


A uniform magnetic field is applied in $-\hat{k}$.

$$\vec{\tau} = \vec{\mu}_B \times \vec{B} \quad \text{r.h.r}$$

$\vec{\tau}$ direction $-\hat{i}$ so $\Delta\vec{L}$ is in $-\hat{i}$

Before $\vec{B}$:



After $\vec{B}$: