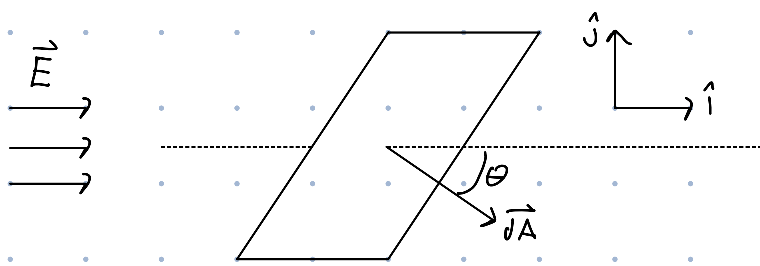
<u>Property of conductors</u>

1) $E$ field is zero inside even if there's an outside electric field
   - Charges move to ensure this happens.
2) Charge always goes to the surface.
3) The $E$ field at the surface is always $\perp$ to the surface.

<u>Flux</u> - Measures how much a field passes through a surface area.

$$\boxed{\Phi = \int \vec{E} \cdot \underbrace{\vec{dA}}_{\text{Surface area.}}}$$

Ex: Find the flux through an area




Polar form:

$$\Phi = \int \vec{E} \cdot \vec{dA}$$

$$= \int E\,dA\cos\theta$$

$$= E\cos\theta \int dA$$

$$= E\cos\theta\, A$$

Cartesian form:

$$\Phi = \int \vec{E} \cdot \vec{dA}$$

$$= \int (E)(dA\cos\theta) + (0)(-dA\sin\theta)$$

$$= \int E\,dA\cos\theta = E\cos\theta \int dA$$

$$= E\cos\theta\, A$$

$$\vec{E} = E\hat{i}$$

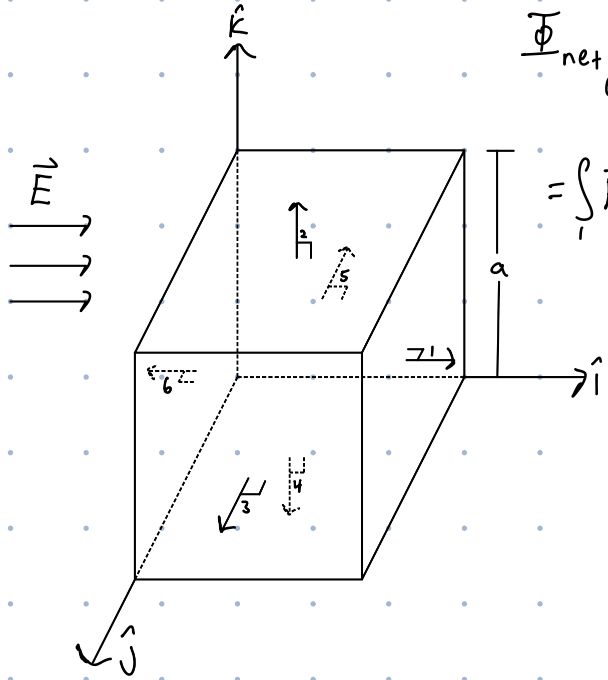
$$\vec{dA} = dA_x\hat{i} - dA_y\hat{j}$$

$$= dA\cos\theta\,\hat{i} - dA\sin\theta\,\hat{j}$$

# Remember the cube

The net flux inside a closed surface (defines a volume) is always $\emptyset$ if the closed surface contains no net charge.

$\vec{dA}$ always points away from the enclosed volume.




$$\Phi_{net\text{ through the cube}} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6$$

$$= \int_1 \vec{E}\cdot\vec{dA} + \underbrace{\cancel{\int_2 \vec{E}\cdot\vec{dA}}}_{\emptyset\perp} + \underbrace{\cancel{\int_3 \vec{E}\cdot\vec{dA}}}_{\emptyset\perp} + \underbrace{\cancel{\int_4 \vec{E}\cdot\vec{dA}}}_{\emptyset\perp} + \underbrace{\cancel{\int_5 \vec{E}\cdot\vec{dA}}}_{\emptyset\perp} + \int_6 \vec{E}\cdot\vec{dA}$$

$$= Ea^2\cos\emptyset + Ea^2\cos 180^\circ$$

$$= Ea^2 - Ea^2 = \emptyset$$

# Gauss's Law for Electric Fields

Used to find $E$ fields quickly when there's a high symetry of charge.

1) $\Phi_{net} = \dfrac{q_{net}}{\varepsilon_0}$ ← Charge enclosed by gaussian surface. / ← Epsilon naught. Permitivity of free space.

Closed Surface

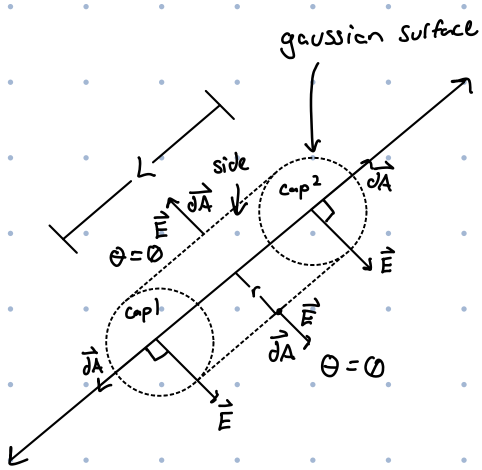
2) $\oint \vec{E}\cdot\vec{dA} = \dfrac{q_{net}}{\varepsilon_0}$

3) $\oint \vec{E}\cdot\vec{dA} = \dfrac{\int \rho \, dVol}{\varepsilon_0}$

Ex: Infinite line of charge. What's $E(r)$? You're given linear charge density ($\lambda$).

$\lambda = \frac{Q}{L}$, but since the line in infinite, $Q$ and $L$ are infinite.

Use a gaussian surface that gives an easy dot product everywhere over the surface.



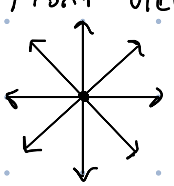

$$\Phi_{net} = \oint \vec{E} \cdot \vec{dA} = \frac{q_{net}}{\varepsilon_0}$$

$$= \overset{0 \perp}{\cancel{\int_{cap1} \vec{E} \cdot \vec{dA}}} + \int_{side} \vec{E} \cdot \vec{dA} + \overset{0 \perp}{\cancel{\int_{cap2} \vec{E} \cdot \vec{dA}}} = \frac{q_{net}}{\varepsilon_0}$$

$$= \int E \, dA \cos\theta = \frac{q_{net}}{\varepsilon_0}$$

E is constant at a given r.

Front view



$$= E \int dA = \frac{q_{net}}{\varepsilon_0}$$

$$= E \underbrace{2\pi r L}_{\text{Surface area of side}} = \frac{\lambda L}{\varepsilon_0}$$

$$= E = \frac{\lambda}{2\pi r \varepsilon_0}$$

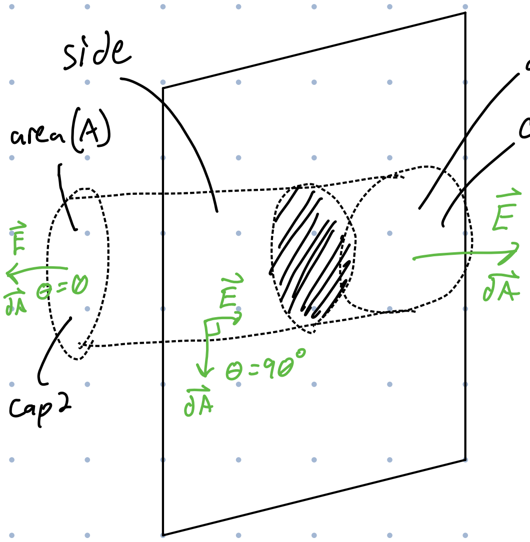
This doesn't work for a finite length line.

When to use gauss's law

- Structure of $E$ field
- Easy dot products everywhere
  $\vec{E}$ direction.

Ex: $\infty$ sheet and $\sigma$ given. $|\vec{E}| = ?$

↑ has to be $\infty$ or else $E$ isn't constant @ the edges.




$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$

$$\theta = 0 \quad \int_{cap1} \vec{E} \cdot d\vec{A} + \cancel{\int_{side} \vec{E} \cdot d\vec{A}}^{\,0} + \int_{cap2} \vec{E} \cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$

$$\int_{cap1} E\,dA \cancelto{1}{\cos\theta} + \int_{cap2} E\,dA \cancelto{1}{\cos\theta}$$

E is uniform

$$E \int_{cap1} dA + E \int_{cap2} dA$$

$$EA + EA = \frac{q_{net}}{\varepsilon_0}$$

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

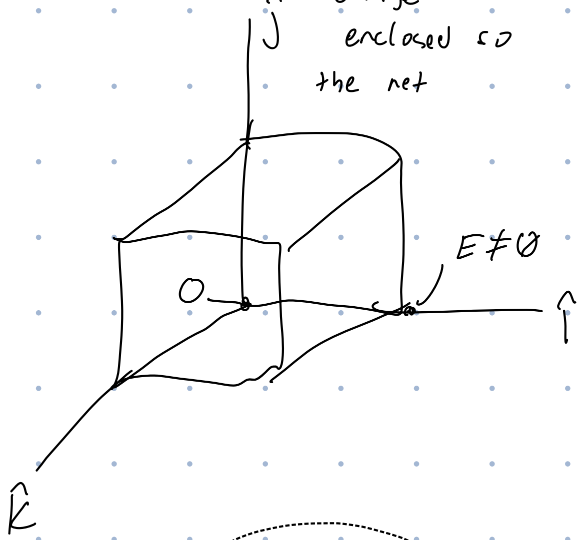
$$\boxed{K = \frac{1}{4\pi\varepsilon_0}}$$

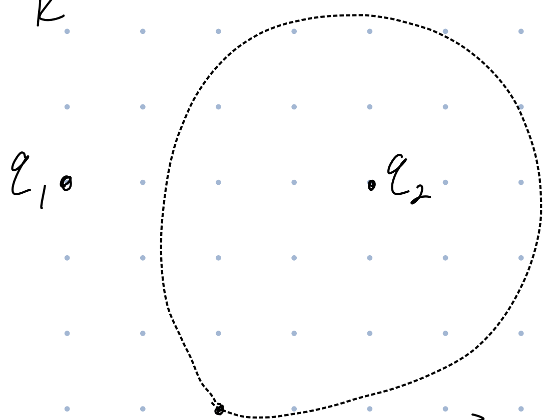
* The gaussian surface can be $\infty$, that's why a finite line doesn't work. When you get to the edges, the e field will pass through the sides that you don't want.

$\vec{E} = Cx^2\hat{i}$ can't exist in nature because there's no charge enclosed so the net

@ O $E=0$ since $x=0$

So $\Phi_{net} \neq 0$ but $q_{net} = 0$




$E = 0$?

From $q_2$ or $\boxed{\text{both}}$? All sources

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\oint (\vec{E_1} + \vec{E_2}) \cdot d\vec{A}$$

0 remember the cube

$$\cancel{\oint \vec{E_1} \cdot d\vec{A}} + \oint \vec{E_2} \cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$

$q_1$ is outside the guassian sphere

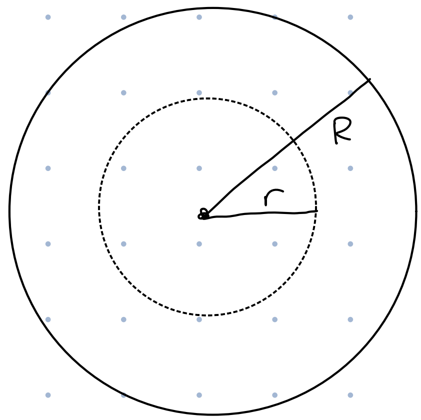
## Guass's law 3 step checklist

3 ways $\Phi_{net} = \oint \vec{E} \cdot d\vec{A}$ can be 0

1) $E = 0$
2) $EA > 0$ and $EA < 0$ and they add to 0 — Remember the cube.
3) $\vec{E} \cdot d\vec{A} = 0$ if $\vec{E} \perp d\vec{A}$

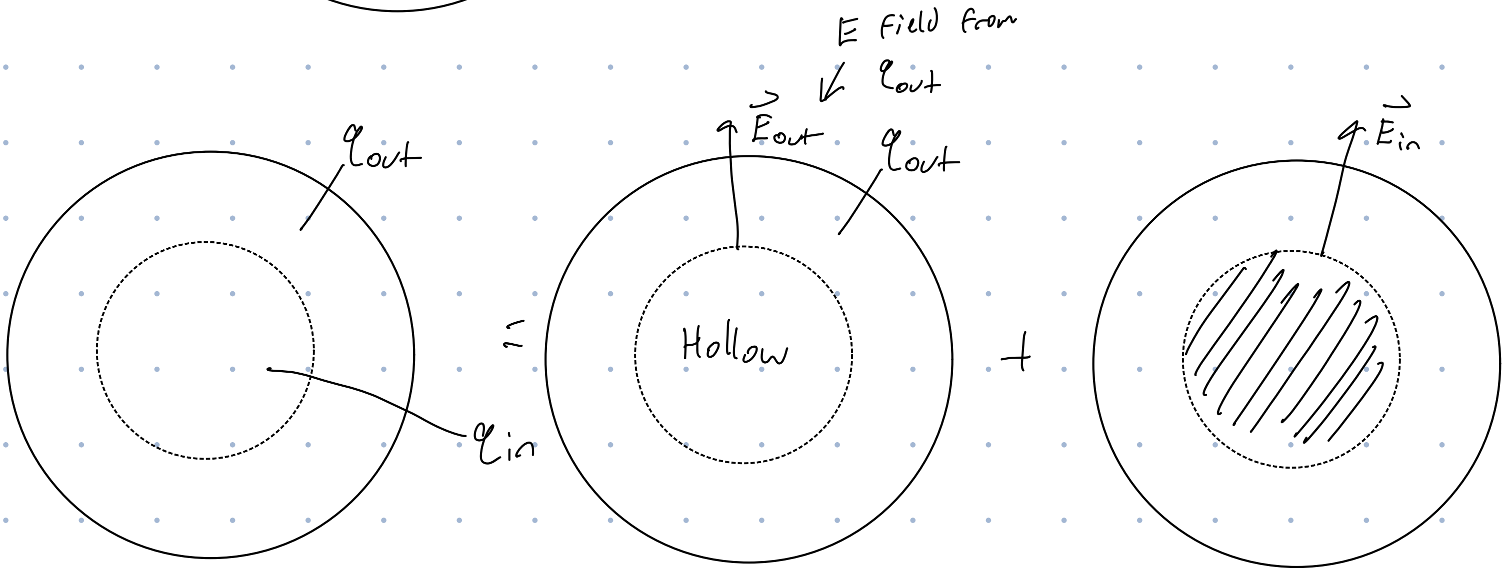
Only find E where the gaussian surface is.

Ex:




$\underline{E(r < R) = ?}$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$




$\vec{E}_{out}$ might not be $\emptyset$

$$\oint \left(\vec{E}_{in} + \vec{E}_{out}\right) \cdot d\vec{A}$$

$$= \oint \vec{E}_{in} \cdot d\vec{A} + \overset{\emptyset}{\cancel{\oint \vec{E}_{out} \cdot d\vec{A}}}$$

why? checklist.

3) By symetry

$d\vec{A}$ cannot be $\perp$ to $\vec{E}_{out}$

2) No way $\vec{E}_{out} \parallel d\vec{A}$ and

$\vec{E}_{out}$ anti-$\parallel$ to $d\vec{A}$

$\therefore E_{out} = \emptyset$

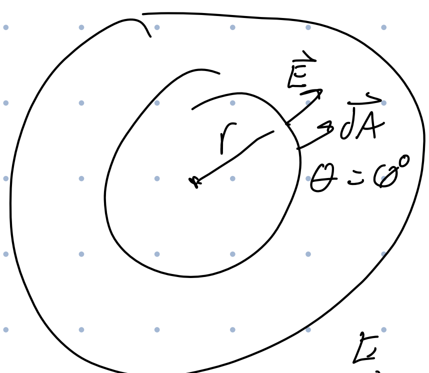
$$= \oint E_{in}\, dA \cancel{\cos\theta}^{\,1}$$

E constant at a given r

$$= E_{in} \oint dA = \frac{\frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$= E_{in} 4\pi r^2 =$$

$$E_{in} = \frac{\rho \frac{4}{3}\pi r}{\varepsilon_0}$$




$$\oint \vec{E} \cdot \vec{dA} = \frac{q_{net}}{\varepsilon_0}$$

$$\oint E\, dA \cancel{\cos\theta}^{\,1} = \frac{q_{net}}{\varepsilon_0}$$

E constant at a given r

$$E \oint dA = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E \cancel{4\pi} \cancel{r^2} = \frac{\rho \cancel{4\pi} r^{\cancel{3}}}{3\varepsilon_0}$$

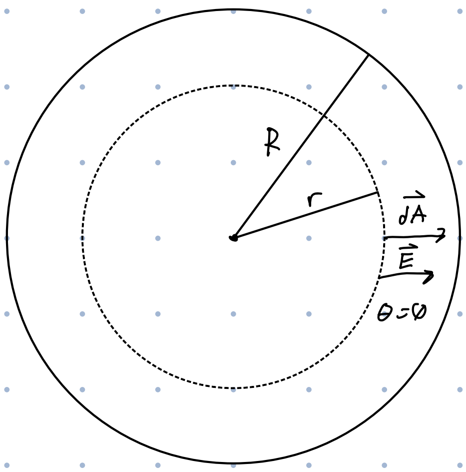
$$E = \frac{\rho r}{3\varepsilon_0}$$

Non uniform density

$$q_{net} = \int \rho\, dVol = \int \rho 4\pi r^2 dr$$

$$dVol = A\, dr$$
$$= 4\pi r^2 dr$$

## Full explination of E-field in dielectric sphere with $\rho$ charge density




$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$

$$\oint \left(\vec{E}_{in}+\vec{E}_{out}\right)\cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$

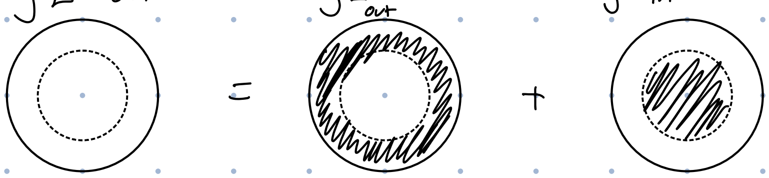
$$\oint \vec{E}_{in}\cdot d\vec{A} + \oint \overbrace{\cancel{\vec{E}_{out}}}^{0 \text{ bc } q_{net\,out}=0}\cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$

$\downarrow$

### Guass's law 3 step checklist

3)

$\oint \vec{E}\cdot d\vec{A}$ $=$ $\oint \vec{E}_{out}\cdot d\vec{A}$ $+$ $\oint \vec{E}_{in}\cdot d\vec{A}$

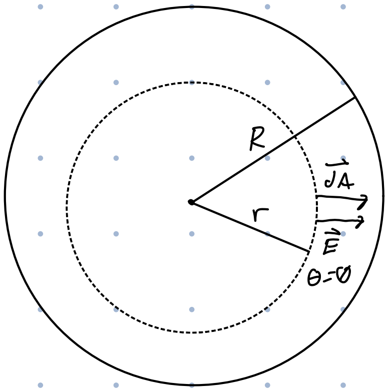


Net flux through the guassian surface

Net flux through the guassian surface from outside charge sources

Net flux through the guassian surface from inside charge sources

## Prove that inside a hollow sphere $E = \emptyset$




$$\oint \vec{E} \cdot \vec{dA} = \frac{q_{net}}{\varepsilon_0}$$

$$\oint \vec{E} \cdot \vec{dA} = \frac{\emptyset}{\varepsilon_0}$$

## Gauss's law 3 step checklist

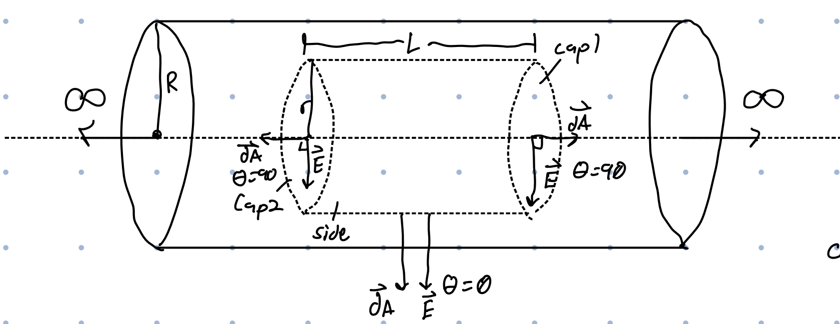
By spherical symetry $\vec{E} \not\parallel \vec{dA}$

~~3)~~ It's impossible for the E-field produced by a sphere to be perpendicular to the surface of the guassian sphere inside.

~~2)~~ It's impossible for the E-field produced by a Sphere to be uniform in 1 direction.

$\therefore E = \emptyset$

## Prove that inside an infinite hollow cylinder $E = \emptyset$




$$\oint \vec{E} \cdot \vec{dA} = \frac{q_{net}}{\varepsilon_0}$$

$$\cancel{\int_{cap1} \overset{\emptyset \perp}{\vec{E} \cdot \vec{dA}}} + \int_{side} \vec{E} \cdot \vec{dA} + \cancel{\int_{cap2} \overset{\emptyset \perp}{\vec{E} \cdot \vec{dA}}} = \frac{\emptyset}{\varepsilon_0}$$

$$\int_{side} E\, dA \cancel{\cos \emptyset}^{1} = \emptyset$$

## Guass's law 3 step checklist

$\cancel{3})$ $\theta = \emptyset$ so $\vec{E}$ and $\vec{dA}$ aren't $\perp$

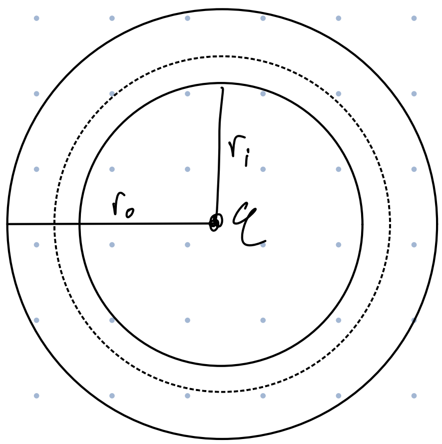
$\cancel{2})$ It's impossible for a hollow cylinder to produce a uniform E-field in one direction

It's not "remember the cube"

$\therefore E = \emptyset$

Ex: Thicc shell conductor no charge

Find $\sigma$ on the inner surface.




$$\oint \cancel{\vec{E}} \cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$

$\emptyset$

by propert of conductors $\emptyset = \dfrac{q_{net}}{\varepsilon_0}$ $\therefore$ $q_{net} = \emptyset$

$q_{net} = {}^{+}q + q_{\text{inner surface}}$

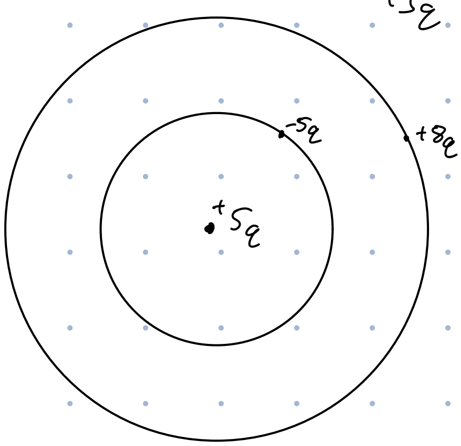
$0 = {}^{+}q + q_{\text{inner surface}}$

$q_{\text{inner surface}} = -q$

$$\sigma = \frac{Q}{A} = \boxed{\frac{-q}{4\pi r_i^2}}$$

---

+3q charged conductor



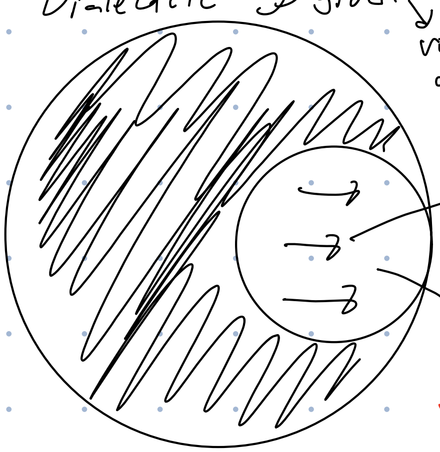

Property of conductors

$E_{inside} = \emptyset$

$q_{net} = \emptyset$ $\therefore$ $q_{\text{inner surface}} = -q_{inside}$

---

Dialectric $\rho$ given → volume so not a conductor. Conductor can't have.

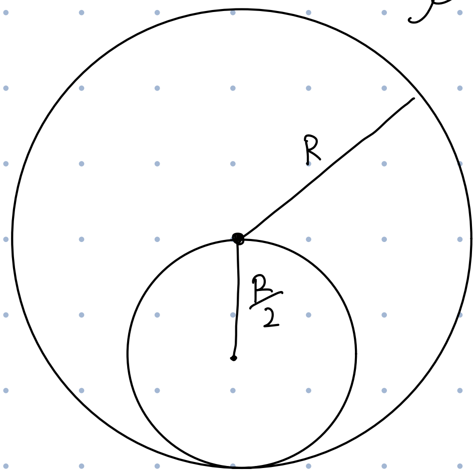


Hollow

Uniform $\vec{E}$

Can use kinematic equations

$\rho$ given

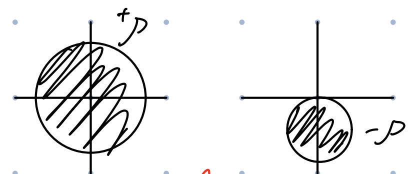



$\vec{E} = ?$ inside the hollow

$$E_{w/\ hollow}\left(r=\frac{R}{2}\right) = E_{w/out\ hollow}\left(r=\frac{R}{2}\right) + E_{hollow\ w/\ -\rho}(r=0)\ \text{(POS)}$$

$$\vec{E} = \vec{E}_{+\rho} + \vec{E}_{-\rho}\ \text{(POS)}$$

$E(r=0) = ?$




0 sym

$$\vec{E} = \cancel{\vec{E}_{+\rho}} + \vec{E}_{-\rho}\ \text{(POS)}$$

$$= \frac{kq}{\left(\frac{R}{2}\right)^2} = \frac{k\left(-\rho \frac{4}{3}\pi \left(\frac{R}{2}\right)^3\right)}{\left(\frac{R}{2}\right)^2}$$

Can treat as a point mass

$E\left(r=\frac{R}{2}\right) = ?$

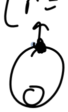
0 sym

$$\vec{E} = \vec{E}_{+\rho} + \cancel{\vec{E}_{-\rho}}\ \text{(POS)}$$

= ~~Gauss's law~~

$\frac{kq}{r^2} = E$ spherical distribution so $E_{at} = 0$

$E(r=R)$ @ top



$$\vec{E} = \vec{E}_{+\rho} + \vec{E}_{-\rho}$$

= No need for gauss's law (Point masses)

How much more $-\rho$ for E @ top $= 0$