Faraday's law

- A changing magnetic flux will induce an E-field.

Induced Emf in units of volts.

$$\boxed{\mathcal{E}_{mf} = -\frac{d\Phi_B}{dt}}$$

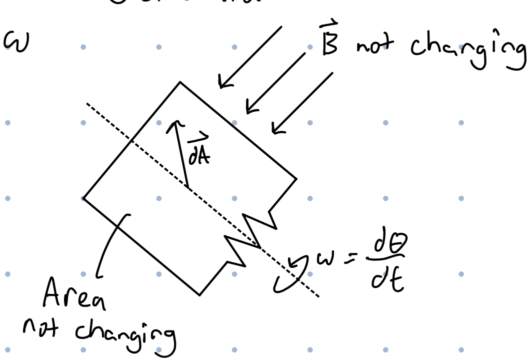
$$\boxed{\mathcal{E}_{mf} = -\frac{d\int \vec{B}\cdot d\vec{A}}{dt}}$$

Lenz's Law (why the minus sign).

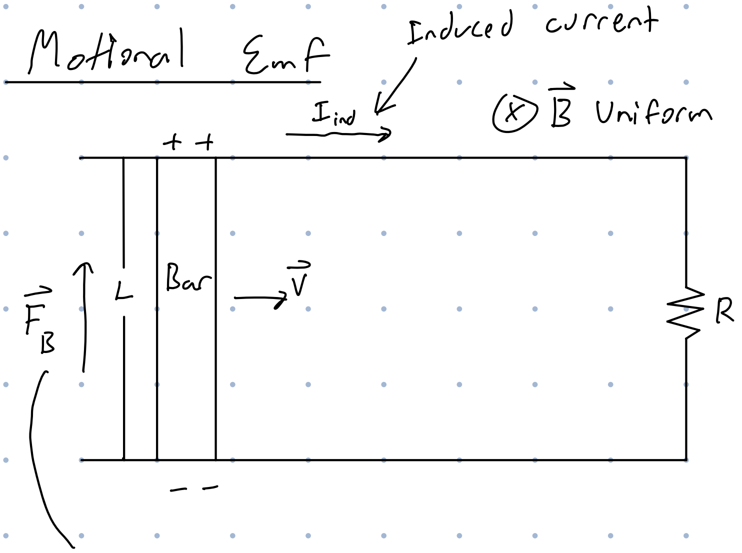
$$\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d\int \vec{B}\cdot d\vec{A}}{dt}}$$

- This E-field is non-conservative, unlike from a source charge.
  - $q\oint \vec{E}\cdot d\vec{l} = 0$ Work done by E-field/Coulomb's force from a source charge over an enclosed loop.
- This E-field has to form closed loops.
- 3 ways magnetic flux (Φ_B) can change over time.
  - $\frac{dB}{dt}$
  - $\frac{dA}{dt}$
  - $\frac{d\theta}{dt} = \omega$

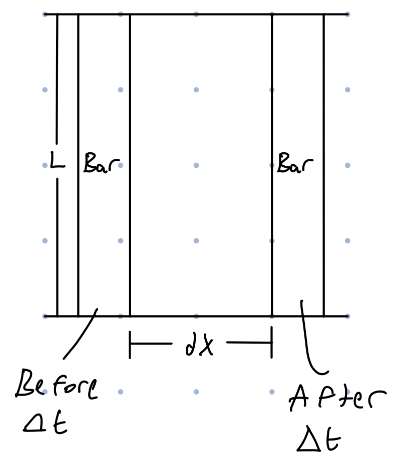
Generator:



Motional Emf



$\vec{F}_B = q(\vec{v} \times \vec{B})$



$dA = dx \cdot L$

$V = \frac{dx}{dt} \rightarrow dx = V \cdot dt$

$dA = V \cdot dt \cdot L$

$A = \int dA = \int vL\,dt = vLt$

$$\left|\mathcal{E}mf\right| = \left|-\frac{d\Phi_B}{dt}\right|$$

$$= \frac{d\int \vec{B}\cdot d\vec{A}}{dt}$$

$$= \frac{d\int B\,dA\,\cancelto{1}{\cos\theta}}{dt}$$

$$= \frac{dBA}{dt}$$

Area is getting smaller

$$= B\frac{dA}{dt}$$

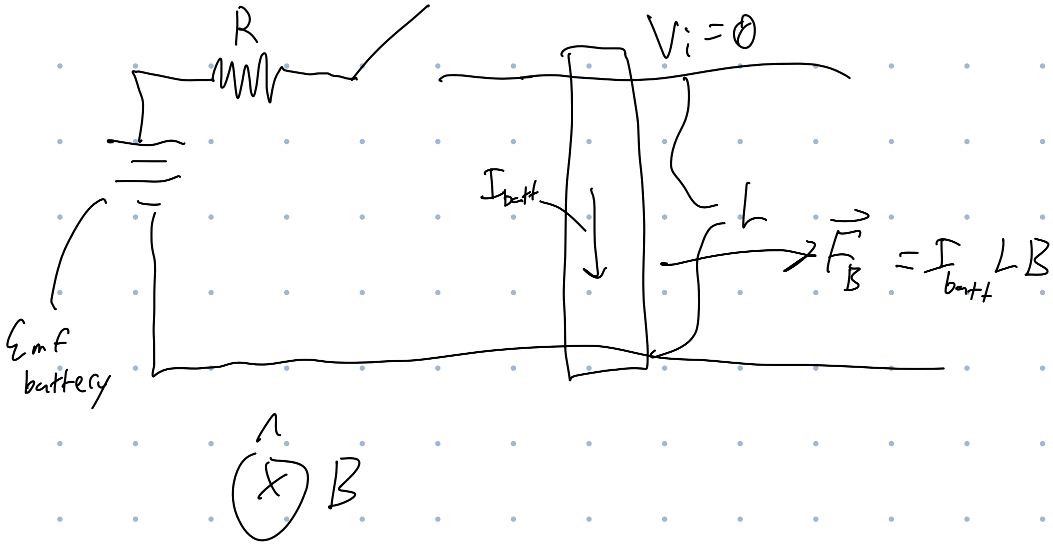
$$= B\frac{d(vLt)}{dt}$$

$$\boxed{\mathcal{E}mf = BvL}$$

How to know direction of $I_{induced}$?

Quick way: $\vec{F}_B = q(\vec{v} \times \vec{B})$ and $\vec{F}_B$ creates a voltage on the bar which can be used to find the direction of I_{ind}.

Lenz's law: Area $\downarrow \rightarrow \Phi_B \downarrow \rightarrow$ Nature wants to keep Φ_B the same
$\rightarrow \uparrow \Phi_B$ from I_{ind} $\rightarrow \uparrow B$ (can't $\uparrow\theta$ or $\uparrow A$)
$\rightarrow$ I clockwize direction to produce an $\uparrow B$ from 2 step r.h.r



I_{batt} doesn't change

$\uparrow I_{ind}$

$\downarrow$

Back Emf.

- Close switch $\rightarrow$ I down through bar $\rightarrow \vec{F}_B = I d\vec{\ell} \times \vec{B}$

  $\rightarrow$ to the right.

- Bar starts to move $\rightarrow$ Induced Emf $\rightarrow \downarrow$ current.

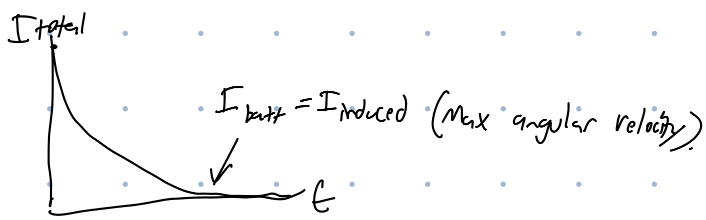
Find $v(t) = ?$
Don't use kinematics.

$$I_{total} = I_{batt} - I_{ind} = \frac{Emf_{batt}}{R} - \frac{BLv}{R}$$

b/c increasing area

Flux $\uparrow$! It wants to keep the same flux. How to make the Flux smaller?
Decrease $\vec{B}$.
Induced $\vec{B}$ in $\odot$

Induced Emf sets the max RPM for motors.



System: Rod

$$\vec{F}_{net} = m\vec{a}$$

$$I_{total} L B = m \frac{dv}{dt}$$

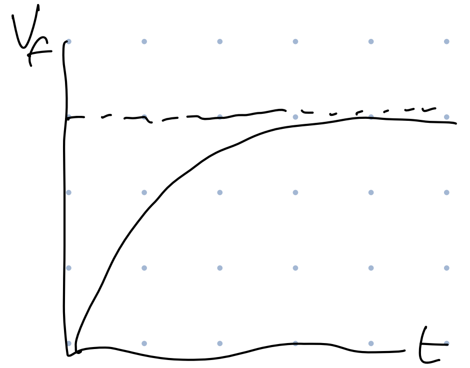
$\downarrow$ separate & integrate

- Put a load on motor $\rightarrow$ lower $\omega \rightarrow$ lower induced Emf $\rightarrow \uparrow I_{total}$

  $\downarrow$

  Burn out motor.

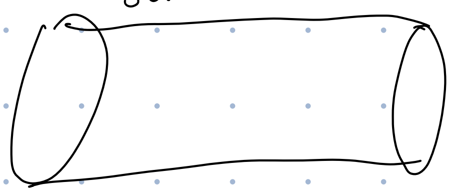
You don't need external $\vec{B}$?



Motional Emf

$\mathcal{E}_{mf} = BLV$

Soloroid

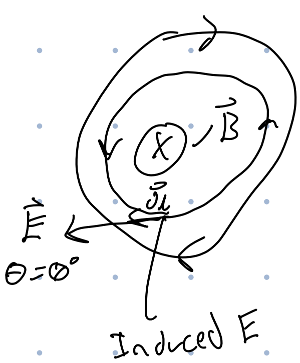


$\frac{dB}{dt}$ bc $\frac{dI}{dt}$

B inside $= \mu_0 n I$

$B_{outside} = \varnothing$

$$\frac{dB}{dt} = \mu_0 n \frac{dI}{dt}$$



E Always form closed loops

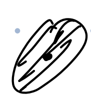
$$\left|\oint \vec{E} \cdot \vec{dl}\right| = \left|-\frac{d\Phi_B}{dt}\right|$$

$E(r) = ?$

Lenz's law:
$B\uparrow \rightarrow \Phi_B \uparrow \rightarrow$ Want to $\downarrow \Phi_B \rightarrow \downarrow B \rightarrow I_{ind} \circlearrowleft \rightarrow E \circlearrowleft$

$$\oint \vec{E} \cdot \vec{dl} = \frac{dBA}{dt}$$

Area



$$\oint E \cdot dl \cdot \cancel{\cos 0}^{1} = A \frac{dB}{dt}$$

$$= A \mu_0 n \frac{dI}{dt}$$

Constant at a given r

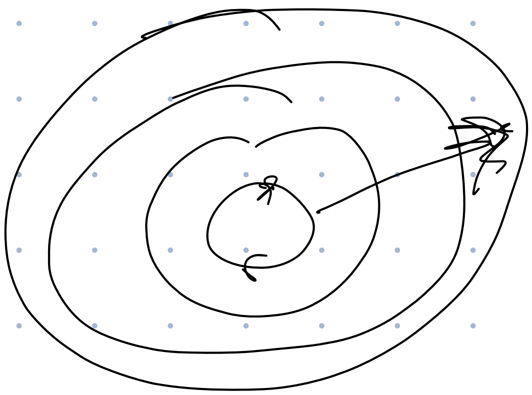
$$E \oint dl$$

$$= \pi r^2 \mu_0 n \frac{dI}{dt}$$

$$E\, 2\cancel{\pi}\cancel{r} = \cancel{\pi} r^{\cancel{2}} \mu_0 n \frac{dI}{dt}$$

$$E = \frac{\mu_0 n \frac{dI}{dt} r}{2}$$

There's induced E-field outside the



decreasing E

How is magnetic flux changing?

E(r) outside?