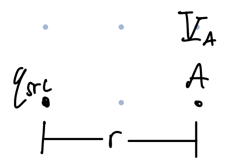
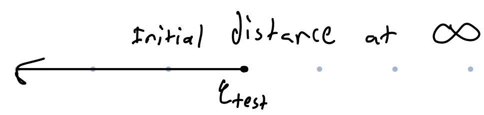
Electric potential (V)

$$\boxed{V = \frac{W}{q_{test}}}$$

Work done by moving a test charge (q_{test}) from ∞ to the point in question at constant speed ($\Delta KE = 0$ / $\Delta V = \emptyset$).





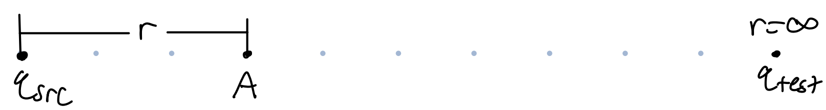
- As q_{test} gets closer, the force increases, and thus you have to do more work.
- V is a scalar field with units $\frac{\text{Joules}}{\text{Coulombs}}$
- A positive test charge will move from high V to low V.

This is a useful modle when you have a lot of src charges.

Voltage - ΔV between two points. Volts $= \frac{\text{Joules}}{\text{Coulomb}}$

Current - Motion of charge that's caused by voltage.

V_A when $V(r=\infty)=\emptyset$



$$W_{\substack{me \\ \infty\to r}}(\Delta KE=\emptyset) = \int_\infty^r \vec{F}_{me}\cdot\vec{d\ell} = -\int_\infty^r \vec{F}_e\cdot\vec{d\ell} = -\int_\infty^r \frac{k q_{src} q_{test}}{r^2}\,\hat{r}\cdot\vec{d\ell}$$

Cartesian form: $\vec{d\ell} = dx\hat{\imath} + dy\hat{\jmath}$

Polar form: $\vec{d\ell} = dr\hat{r} + r d\theta\hat{\theta}$

Since $\vec{F}_e$ is a conservatice force, it's path independent, so we can always choose a radial path. $\vec{d\ell} = dr\hat{r} + \cancel{rd\theta}^{\emptyset}\hat{\theta}$

$\hat{r}\cdot\hat{r}=1$

$$-\int_\infty^r \frac{k q_{src} q_{test}}{r^2}\,\hat{r}\cdot dr\hat{r} = -\int_\infty^r \frac{k q_{src} q_{test}}{r^2}\,dr$$

$$= -k q_{src} q_{test}\int_\infty^r \frac{1}{r^2}dr = -k q_{src} q_{test}\left[-\frac{1}{r}\right]_\infty^r = \frac{k q_{src} q_{test}}{r}$$

$$W_{\substack{me \\ \infty\to r}}(\Delta KE=\emptyset) = \Delta\,\text{Energy}$$

$$= \cancel{\Delta K}^{\emptyset} + \Delta U$$

$$= \boxed{\Delta U_{\infty\to r} = U = \frac{k q_{src} q_{test}}{r}}$$

$$V_A = \frac{W_{\substack{me \\ \infty\to r}}(\Delta KE=\emptyset)}{q_{test}} = \frac{k q_{src} q_{test}}{r q_{test}}$$

$$\boxed{V_A = \frac{k q_{src}}{r} \quad \text{when } V(r=\infty)=\emptyset}$$

$$\boxed{\Delta U = q_{test}\,\Delta V}$$

Energy

Work changes energy.

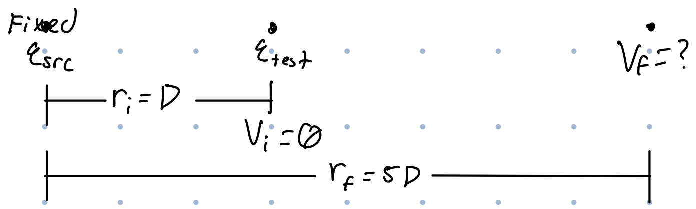
$$\boxed{W_{\substack{net\\ext}} = \Delta \text{ Energy}}$$ Work energy theorem

$$= \Delta K + \Delta U$$

ΔK: kinetic; ΔU: Potential

- Kinetic energy is the energy a body has because it's moving $\left(\frac{1}{2}mV^2\right)$.

Ex: Find V_f at $r = 5D$



- You can't use kinematic equations because a isn't constant.

System: q_{src} & q_{test}

0 ($\vec{dl} = 0$ for fixed force)

$$\cancel{W_{\substack{net\\ext}}} = \Delta \text{Energy}$$

$$0 = \Delta K + \Delta U$$

$0\ (v_i = 0)$

$$0 = (K_f - \cancel{K_i}) + (U_f - U_i)$$

$$0 = \frac{1}{2}mV_f^2 + \frac{kq_{src}q_{test}}{5D} - \frac{kq_{src}q_{test}}{D}$$

$$0 = \frac{1}{2}mV_f^2 + \frac{kq_{src}q_{test}}{5D} - \frac{5kq_{src}q_{test}}{5D}$$

$$\frac{4kq_{src}q_{test}}{5D} = \frac{1}{2}mV_f^2$$

$$V_f = \sqrt{\frac{8kq_{src}q_{test}}{5Dm}}$$

System: q_{test}

$$W_{\substack{net\\ext}} = \Delta \text{Energy}$$

0 (No U in the system)

$$W_{src} = \Delta K + \cancel{\Delta U}$$

$0\ (v_i = 0)$

$$-q_{test}\Delta V = (K_f - \cancel{K_i})$$

$$-q_{test}(V_f - V_i) = \frac{1}{2}mV_f^2$$

$$-q_{test}\left(\frac{kq_{src}}{5D} - \frac{kq_{src}}{D}\right) = \frac{1}{2}mV_f^2$$

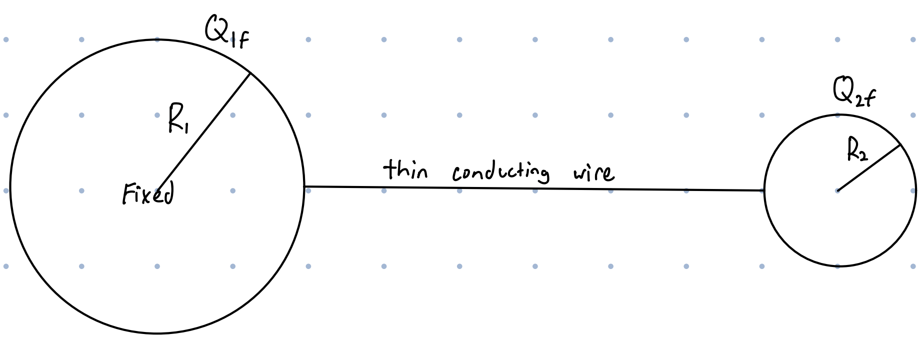
$$-q_{test}\left(-\frac{4kq_{src}}{5D}\right) = \frac{1}{2}mV_f^2$$

$$V_f = \sqrt{\frac{8kq_{src}q_{test}}{5Dm}}$$

ΔV Property of conductors

Before:

After:



What is Q_{1f} and Q_{2f}?

$Q_{1i} + Q_{2i} = Q_{1f} + Q_{2f}$ (Charge is conserved)

$V_{1f} = V_{2f}$ (Property of conductors)

$$\frac{kQ_{1f}}{R_1} = \frac{kQ_{2f}}{R_2}$$

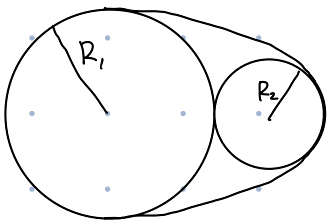
Two unknowns and two equations.

$$Q_{1f} = \frac{R_1(Q_{1i} + Q_{2i})}{R_1 + R_2}$$

$$Q_{2f} = \frac{R_2(Q_{1i} + Q_{2i})}{R_1 + R_2}$$

Remember the egg

σ is not constant over a conducting egg.



$V_1 = V_2$ (Property of conductors) $\quad \sigma_1 = \frac{Q_1}{A_1} = \frac{Q_1}{4\pi R_1^2} \rightarrow Q_1 = \sigma_1 4\pi R_1^2$

$$\frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} \qquad \sigma_2 = \frac{Q_2}{A_2} = \frac{Q_2}{4\pi R_2^2} \rightarrow Q_2 = \sigma_2 4\pi R_2^2$$

$$\frac{k\sigma_1 4\pi R_1^2}{R_1} = \frac{k\sigma_2 4\pi R_2^2}{R_2}$$

$$\sigma_1 R_1 = \sigma_2 R_2$$

More charge on the right then the left.

Ex: When you're charged and you stick your finger out near a surface it creates a spark. More charge is on your finger than body.

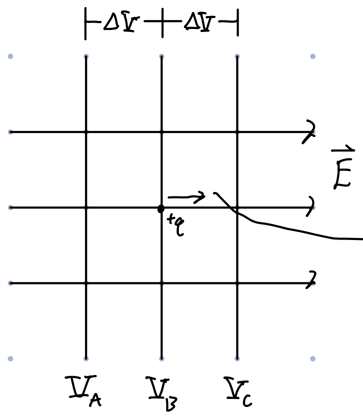
V when $V(r=a)=0$

$$\Delta V_{a\to b} = \frac{W_{me\,a\to b}}{q} = \frac{\int_a^b \vec{F}_{me}\cdot d\vec{\ell}}{q} = \frac{-\int_a^b \vec{F}_e\cdot d\vec{\ell}}{q} = \frac{-\int_a^b q\vec{E}\cdot d\vec{\ell}}{q} = \frac{-q\int_a^b \vec{E}\cdot d\vec{\ell}}{q}$$

$$\boxed{\Delta V_{a\to b} = -\int_a^b \vec{E}\cdot d\vec{\ell}}$$

- Following the E-field decreases V ($-\Delta V$).

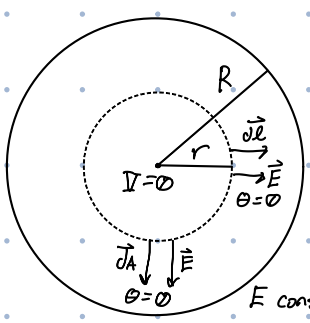
Ex: Uniform E-field



Follows E-field and goes from high (V_B) to low (V_c).

$$\Delta V_{B\to C} = V_C - V_B = \text{negative/decreases}$$

Ex: Find $V(r=R)$ where $V(r=0)=0$.



E constant at a given r.

Gauss's law:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}$$

$$\oint E\,dA\cos\overset{1}{\cancel{0}} = \frac{q_{net}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{net}}{\varepsilon_0}$$

$$E\,4\pi r^2 = \frac{\rho\frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E = \frac{\rho r}{3\varepsilon_0}$$

$d\vec{\ell} = d\vec{r}$ since $\theta = 0$

$$\Delta V = \underbrace{V_f}_{V(r=R)} - \overset{0}{\cancel{V_i}} = -\int_0^R \vec{E}\cdot d\vec{\ell} = -\int_0^R E\,dr\cos\overset{1}{\cancel{0}}$$

$$= -\int_0^R \frac{\rho r}{3\varepsilon_0}\,dr = -\frac{\rho}{3\varepsilon_0}\int_0^R r\,dr$$

$$= -\frac{\rho}{3\varepsilon_0}\left[\frac{1}{2}r^2\right]_0^R = -\frac{\rho R^2}{6\varepsilon_0}$$

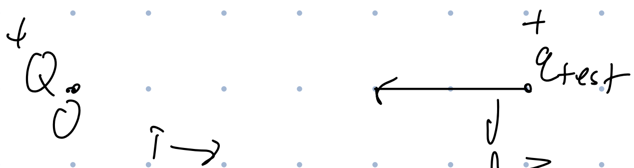
$$k = \frac{1}{4\pi\varepsilon_0} \rightarrow \varepsilon_0 = \frac{1}{4\pi k}$$

$$\rho = \frac{Q}{Vol} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$

$$V(r=R) = -\frac{kQ}{2R} \quad \text{not } \frac{kQ}{R} \text{ when } V(r=\infty)=0$$

$$V_e = \frac{k q_1 q_2}{r}$$ As long as it isn't changing.

$$\vec{dl} = dx\,\hat{i} + dy\,\hat{j}$$



$$W_e = \int_{\infty}^{d} \vec{F}_{Q\to q} \cdot \vec{dl}$$

$$= \int_{\infty}^{d} \frac{kqQ}{x^2}\,\hat{i} \cdot dx\,\hat{i}$$

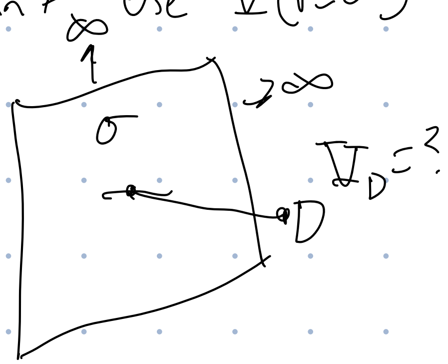
Don't include (←)

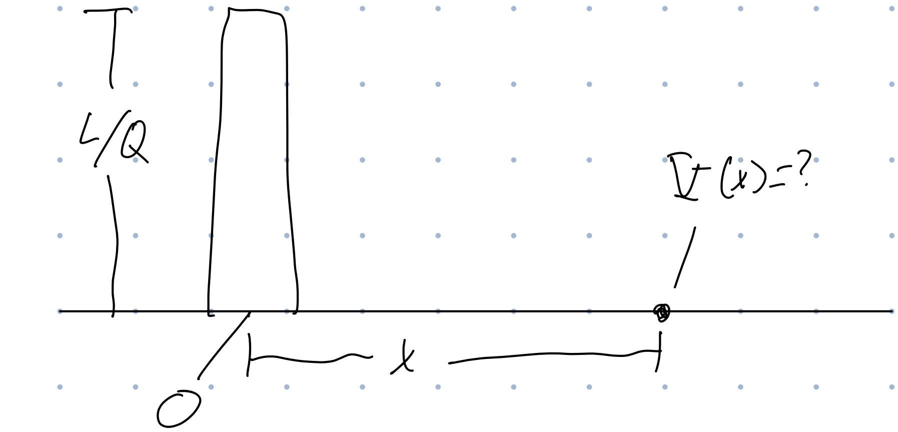
Don't tell integral the direction

ΔV

$$V_f - V_i^{\,0} = -\int_i^f \vec{E} \cdot \vec{dl}$$

Can't use $V(r=\infty)=0$ for ∞ line.



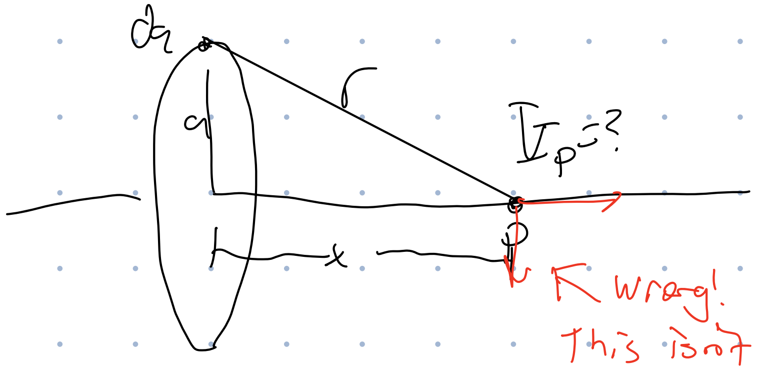


? →

E-field → $-\int \vec{E}\cdot d\vec{l}$

or

$$V = \int dV = \int \frac{k\,dq}{r} \quad (\text{POS})$$



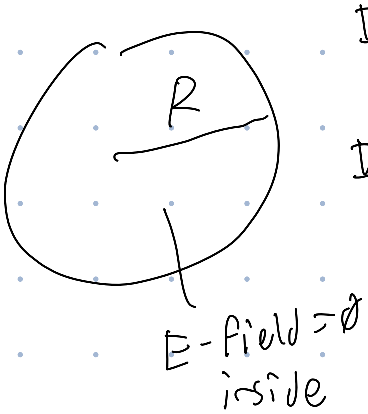
$$V_p = \int dV \quad (\text{POS})$$

$$= \int \frac{k\,dq}{r}$$

$$= \frac{k}{r}\int dq$$

$$= \frac{kQ}{r}$$

Hollow



$$V = \frac{kQ}{R}$$

V_{inside} = constant

bc $\Delta V = -\int \vec{E}\cdot d\vec{l}$, with $\vec{E} = 0$