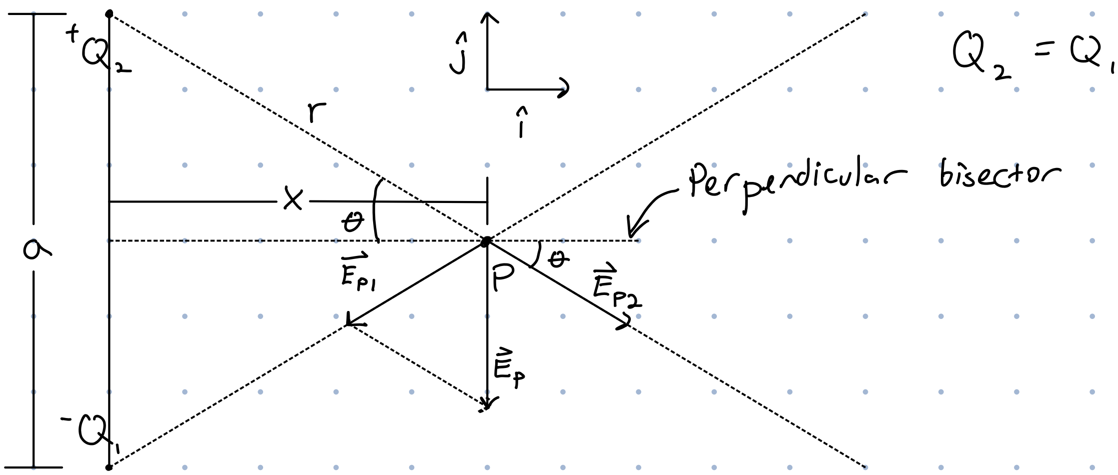
Electric field on the perpendicular bisector of a Dipole

A dipole is two opposite charges of eqcal magnite held at a fixed distance from eachother.



$$\vec{E}_P = \vec{E}_{P1} + \vec{E}_{P2} \quad (POS)$$

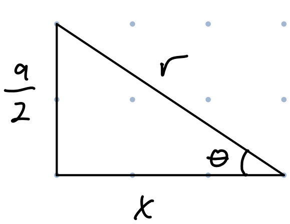
$$= \left(\vec{E}_{P1x} + \vec{E}_{P1y}\right) + \left(\vec{E}_{P2x} + \vec{E}_{P2Y}\right)$$

$$= \overset{0 \text{ symetry}}{\cancel{\left(E_{P1x} + E_{P2x}\right)\hat{i}}} + \left(E_{P1Y} + E_{P2Y}\right)\hat{j}$$

Symetry
$E_{P1Y} = E_{P2Y}$

$$= 2E_{P2Y}\,\hat{j}$$

$$E_{P2Y} = E_{P2} \cdot \sin\theta = \frac{kQ_2}{r^2} \cdot \sin\theta = \frac{kQ_2}{x^2+\left(\frac{a}{2}\right)^2} \cdot \frac{a}{2r}$$



$$r^2 = x^2 + \left(\frac{a}{2}\right)^2$$

$$\sin\theta = \frac{\frac{a}{2}}{r} = \frac{a}{2r}$$

$$= \frac{kQ_2}{\left(x^2+\left(\frac{a}{2}\right)^2\right)} \cdot \frac{a}{2\sqrt{x^2+\left(\frac{a}{2}\right)^2}} = \frac{kQ_2\,a}{2\left(x^2+\left(\frac{a}{2}\right)^2\right)^{3/2}}$$

$$= \frac{KQa}{\left(x^2+\left(\frac{a}{2}\right)^2\right)^{3/2}}\,\hat{j}$$

Torque

What is force? $F_g = \frac{Gm_1 m_2}{r^2}$, $F_e = \frac{Kq_1 q_2}{r^2}$

what does force do? Causes acceleration $\left(\vec{F} = m\vec{a} \text{ and } \frac{d\vec{p}}{dt}\right)$

- Deceleration only when velocity is opposite to acceleration.

What is torque? $\vec{\tau} = \vec{r} \times \vec{F}$

what does torque do? Causes angular acceleration. $\left(\vec{\tau} = I\vec{\alpha} \text{ and } \frac{d\vec{l}}{dt}\right)$

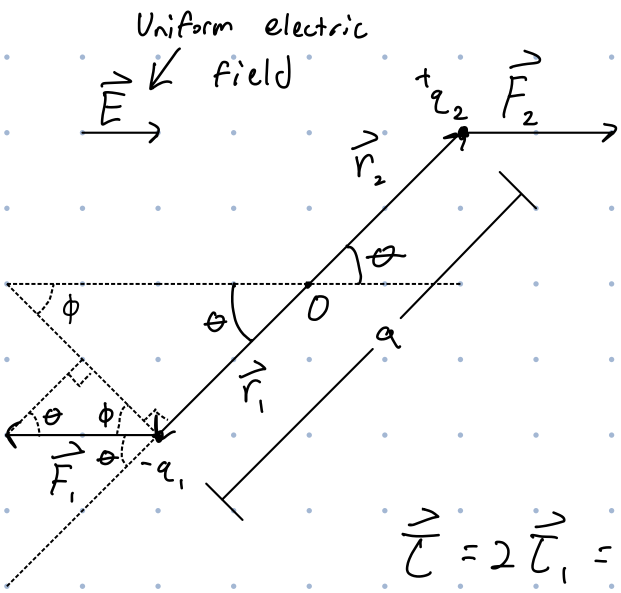
Direction: Right hand rule

1. Align your palm with $\vec{r}$
2. Rotate hand so your fingers curle in the same direction as $\vec{F}$
3. The direction is where your thumb points.

$\odot$ - Out like a tip of an arrow

$\otimes$ - In like the fletching of an arrow

Torque on a dipole from a constant electric field



$\vec{\tau} = \vec{\tau_1} + \vec{\tau_2}$

Symetry

$\vec{\tau_1} = \vec{\tau_2}$

$\vec{\tau_1} = \vec{r_1} \times \vec{F_1}$

$= r_1 F_1 \sin\theta \, \hat{\otimes}$ by r.h.r

$\vec{\tau} = 2\vec{\tau_1} = 2 r_1 F_1 \sin\theta \, \hat{\otimes} = 2 \frac{a}{2} qE \sin\theta \, \hat{\otimes}$

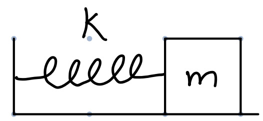
$= aqE \sin\theta \, \hat{\otimes}$

Dipole moment - Vector that describes how strong the dipole is ($\vec{p}$).

$$\vec{\tau} = \vec{p} \times \vec{E} \rightarrow aqE\sin\theta = pE\sin\theta \rightarrow p = aq$$

Period of oscillation - Time for 1 oscillation (T)

Use the analogy of a spring:



$\vec{F} = -k\,\Delta\vec{x}$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

For a dipole: $\vec{\tau} = K\,\Delta\vec{\theta}$ $\qquad T = 2\pi\sqrt{\frac{I}{K}}$

$$I = \underset{\uparrow}{mr^2} + mr^2 = 2m\left(\frac{a}{2}\right)^2 = \frac{1}{2}ma^2 \qquad K = \frac{\tau}{\Delta\theta} = \frac{aqE\sin\theta}{\Delta\theta} \underset{\text{For small } \theta\text{s}}{\approx} aqE$$

Moment of inertia for a point mass

$$T = 2\pi\sqrt{\frac{ma}{2qE}}$$