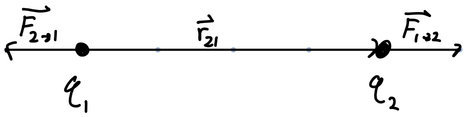
## Coulomb's force law with vectors

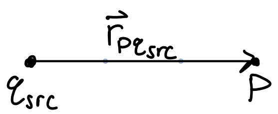



$$\boxed{\vec{F}_{1\to2} = \frac{k q_1 q_2}{r^2}\,\hat{r}_{21}}$$

$\vec{F}_{1\to2}$ = Force on 2 from 1

$\vec{r}_{21}$ = The vector on 2 pulling 1

## Electric field



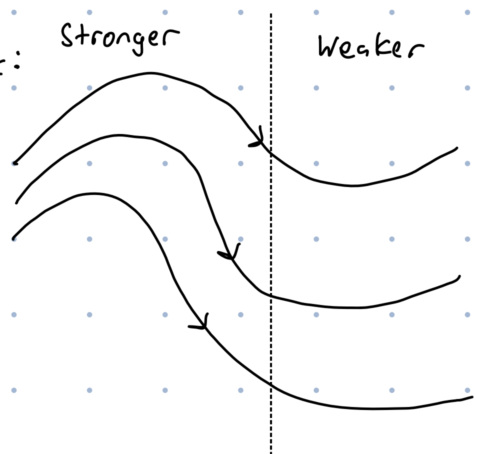

$$\boxed{\vec{E}_P = \frac{k q_{src}}{r^2}\,\hat{r}_{Pq_{src}}}$$

## Electric field lines

Direction is the tangent line.

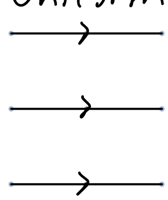
Magnitude is how close the lines are.

Ex:



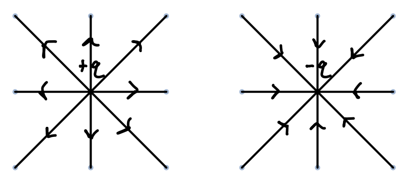

Particles don't always follow the electric field lines.

(1) Uniform

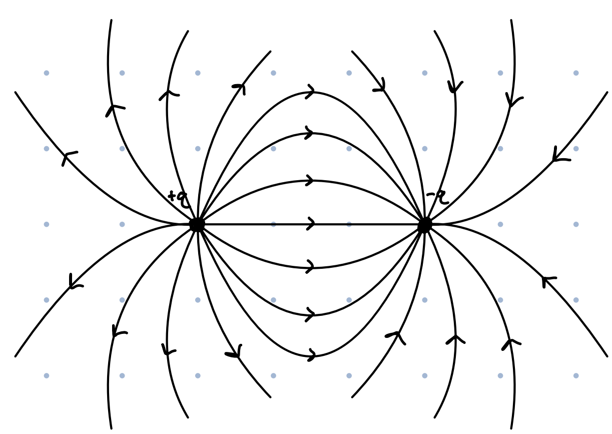


(2) Non-uniform

(A) Radial




(B) Dipole



# Continious distribution of charge with $\hat{r}$

⓪ Draw diagram
- O for origin
- $\hat{\imath}$ and $\hat{\jmath}$
- Draw $dq$, dashed line to point (P) with length $r$, $\hat{r}$, and $d\vec{E}$
- Variable that's changing across the integral

① $\vec{E}_P = \int d\vec{E}$ (POS)

② $= \int \frac{k\,dq}{r_{Pdq}^2} \hat{r}_{Pdq}$ $dq$ creates a $dE$

③ Use density to convert $dq$ into geometry that can be integrated

Linear: $\lambda = \frac{Q}{L} = \frac{dq}{dy} \rightarrow dq = \frac{Q}{L} dy$

Area: $\sigma = \frac{Q}{A} = \frac{dq}{dA}$

Volume: $\rho = \frac{Q}{Vol} = \frac{dq}{dV}$

$$= \int \frac{kQ}{L r_{Pdq}^2} dy\, \hat{r}_{Pdq}$$

④ Convert $\hat{r}_{Pdq}$ to $\hat{\imath}$ and $\hat{\jmath}$

$\vec{r}_{Pdq} = \vec{r}_{PO} + \vec{r}_{Odq}$ Change vector in terms of origin (O).

$= x\hat{\imath} + y\hat{\jmath}$ Use diagram to do vector addition with cartesian coordinates

$r_{Pdq} = \sqrt{x^2 + y^2}$

$$= \int \frac{kQ}{L r_{Pdq}^2} dy\, \hat{r}_{Pdq}$$

$\hat{r}_{Pdq} = \frac{\vec{r}_{Pdq}}{r_{Pdq}} = \frac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2 + y^2}}$

$$= \int \frac{kQ}{L(x^2 + y^2)} dy \frac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2 + y^2}}$$

⑤ Solve the integral with bounds. Answere in variables given.

$$= \frac{kQ}{L} \int_{\emptyset}^{L} \frac{x\hat{i} + y\hat{j}}{(x^2+y^2)^{3/2}}\, dy$$

$$= \frac{kQ}{L} \left[ \int_{\emptyset}^{L} \frac{x\hat{i}}{(x^2+y^2)^{3/2}}\, dy + \int_{\emptyset}^{L} \frac{y\hat{j}}{(x^2+y^2)^{3/2}}\, dy \right]$$

$y = x \tan u$

$u = x^2 + y^2$

$du = 2y\, dy \rightarrow dy = \frac{du}{2y}$

## Continious distribution of charge with symetry

⓪ Draw diagram

- Same as with $\hat{r}$
- Add $\theta$ and $d\vec{E}$ vector components ($E_x$: $d\vec{E_x}$ and $d\vec{E_y}$)

① $\vec{E_P} = \int d\vec{E}$ (POS)

② $= \int (d\vec{E_x} + d\vec{E_y}) = \int d\vec{E_x} + \int d\vec{E_y}$ Break integral into cartesian coordinates.

③ $= \int d\vec{E_x} + \cancel{\int d\vec{E_y}}$ $\emptyset$ symetry Set an integral to $\emptyset$ due to symetry

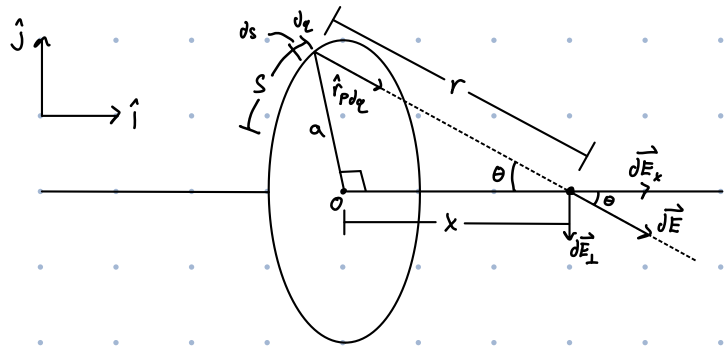
④ $dE_x = dE \cos\theta$ or $\sin\theta$ Convert remaining integral into $d\vec{E}$

$$= \int dE \cos\theta\ \hat{i}$$

⑤ Density

⑥ Solve integral with bounds

# Ring of charge example




$$\vec{E} = \int d\vec{E} \quad (POS)$$

$$= \int d\vec{E_x} + d\vec{E_\perp}$$

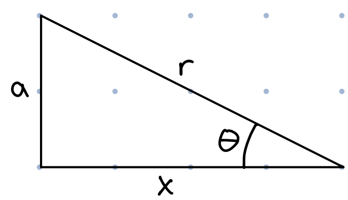
## $d\vec{E_x}$ in terms of $d\vec{E}$

$$dE_x = dE\cos\theta$$

$$= \int d\vec{E_x} + \cancel{\int d\vec{E_\perp}}^{\,0}$$

0 symetry

## $\cos\theta$ in terms of $x$ and $r$




$$\cos\theta = \frac{x}{r}$$

$$= \int dE\cos\theta\,\hat{i}$$

$$= \int \frac{k\,dq}{r^2}\cdot\frac{x}{r}\,\hat{i}$$

## Density

$$\lambda = \frac{Q}{2\pi a} = \frac{dq}{ds} \rightarrow dq = \frac{Q}{2\pi a}\cdot ds$$

$$= \int \frac{xkQ}{2\pi a r^3}\,ds\,\hat{i} = \frac{xkQ}{2\pi a r^3}\,\hat{i}\int_0^{2\pi a} ds$$

$$= \frac{xkQ}{r^3}\,\hat{i}$$

## $r$ in terms of $a$ and $x$

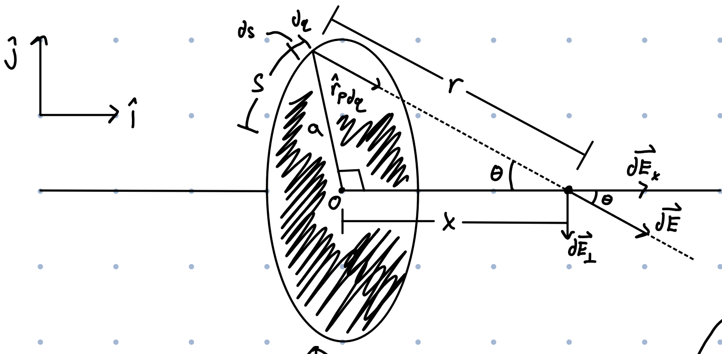
$$r^2 = a^2 + x^2$$

$$r^3 = (a^2 + x^2)^{3/2}$$

$$= \frac{xkQ}{(a^2+x^2)^{3/2}}\,\hat{i}$$

# Circular disk of charge

Conductors only have charge on the outside




No thickness

$$\vec{E}_{ring} = \frac{kxQ}{(x^2+a^2)^{3/2}}\,\hat{i}$$

$$d\vec{E}_{ring} = \frac{kx\,dq}{(x^2+r^2)^{3/2}}\,\hat{i}$$

$$\vec{E} = \int d\vec{E}_{ring} \quad (POS)$$

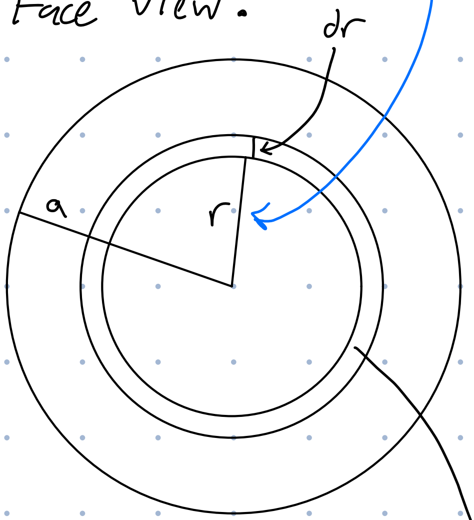
$$= \int \frac{kx\,dq}{(x^2+r^2)^{3/2}}\,\hat{i}$$

$$= \int \frac{kx}{(x^2+r^2)^{3/2}}\left(\frac{2Qr}{a^2}\,dr\right)\hat{i}$$

$$= \frac{2Qkx}{a^2}\,\hat{i}\int_0^a \frac{r}{(x^2+r^2)^{3/2}}\,dr$$

U-sub

Face view:

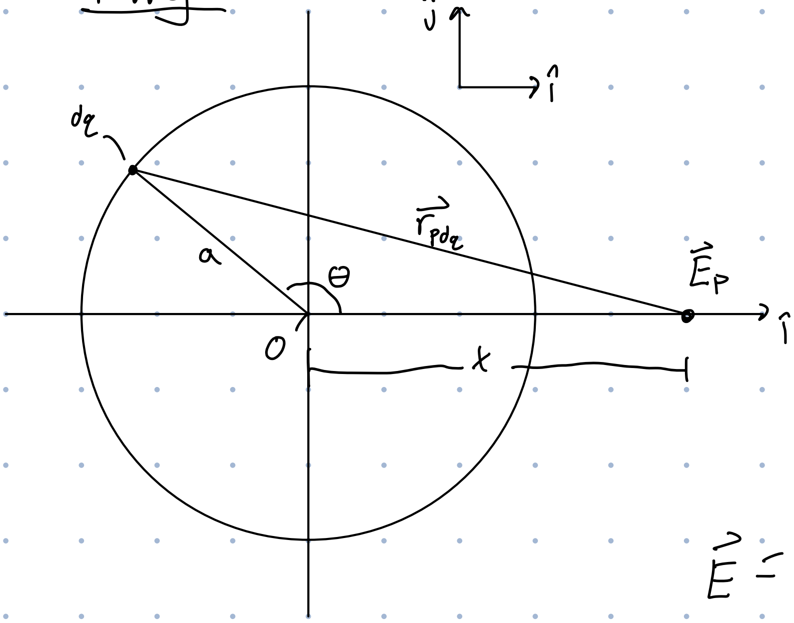



## dq in terms of


$$dA = 2\pi r\,dr$$

$$\sigma = \frac{Q}{A} = \frac{dq}{dA} \rightarrow dq = \frac{Q}{A}\,dA = \frac{Q}{\pi a^2}\cdot 2\pi r\,dr = \frac{2Qr}{a^2}\,dr$$

# Ring




$$\vec{E} = \int d\vec{E} \quad (POS)$$

$$= \int \frac{k\,dq}{r_{pdq}^2}\,\hat{r}_{pdq}$$

## $\hat{r}_{pdq}$ in terms of $\hat{i}$ and $\hat{j}$

$$\vec{r}_{pdq} = \vec{r}_{po} + \vec{r}_{odq}$$

$$= x\hat{i}\ \cancel{x}^{-a}\cos\theta\,\hat{i}\ \cancel{x}^{-a}\sin\theta\,\hat{j}$$

$$= (x - a\cos\theta)\hat{i} - a\sin\theta\,\hat{j}$$

$$\hat{r}_{pdq} = \frac{(x - a\cos\theta)\hat{i} - a\sin\theta\,\hat{j}}{\sqrt{(x - a\cos\theta)^2 + a^2\sin^2\theta}}$$

## $dq$ in terms of $d\theta$

$$\lambda = \frac{Q}{L} = \frac{dq}{ds} \rightarrow dq = \frac{Q}{2\pi a}\cdot a\,d\theta$$

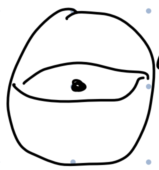
$$L = 2\pi a \qquad\qquad = \frac{Q}{2\pi}\,d\theta$$

$$ds = a\,d\theta$$

$F_{1\to2}$

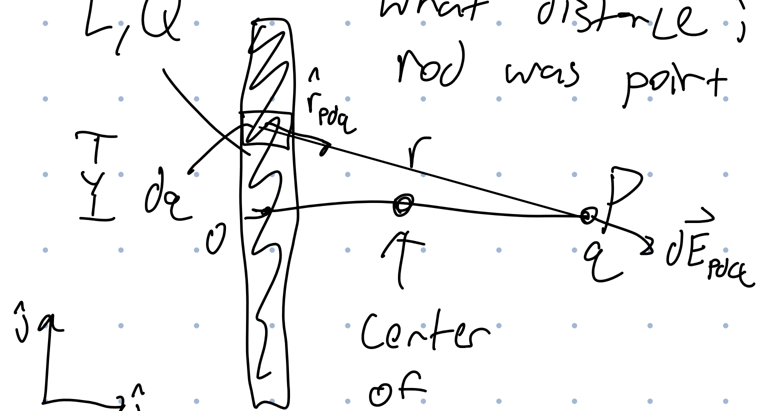
$q_1$ •       • $q_2$

$F = \dfrac{k q_1 q_2}{r^2}$



Only for spheres. You can use center to center.

What distance if rod was point charge.




1. Find $\vec{F}_p$

By newton's 3rd law $\overrightarrow{F_{rod\to p}} = -\vec{F}_{p\to rod}$

$\hat{r}_{pdq}$ in terms of $\hat{i}$ & $\hat{j}$

$$\vec{r}_{pdq} = \overrightarrow{r_{po}} + \overrightarrow{r_{odq}}$$

$$= x\hat{i} - y\hat{j}$$

$$\hat{r}_{pdq} = \frac{\vec{r}}{r} = \frac{x\hat{i} - y\hat{j}}{\sqrt{x^2+y^2}}$$

Better

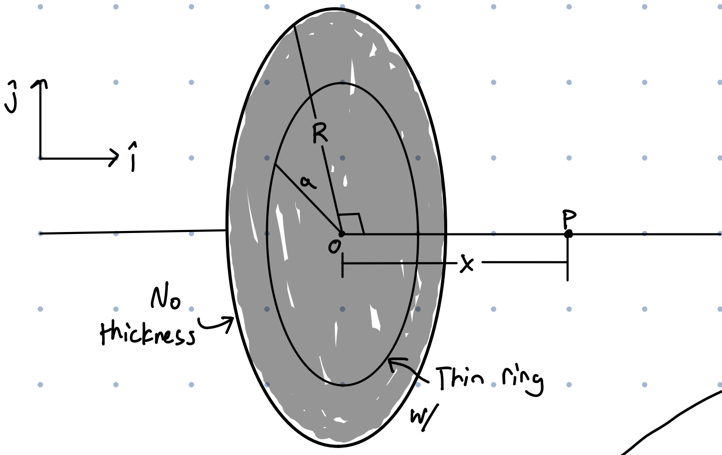
$$\int dF = \int \frac{k\,dq\,q}{r^2} \quad \text{or} \quad \vec{F} = q\vec{E}_{rod}$$

$$\overrightarrow{E_{rod}} = \int d\vec{E}_{rod} \ (POS)$$

$$= \int \frac{k\,dq}{r_{pdq}^2}\hat{r}_{pdq}$$

$$= \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{k\lambda\,dy}{\phantom{x}}$$

## Circular disk of charge




2D: $\vec{E}_P = \int d\vec{E}_{\text{thin ring}}$ (POS)

$$= \int \frac{x\,k\,dq}{(a^2+x^2)^{3/2}}\,\hat{\imath}$$

$$= \int \frac{x\,k}{(a^2+x^2)^{3/2}}\,\frac{2Qa}{R^2}\,da\,\hat{\imath}$$

$$\vec{E}_{\text{thin ring}} = \frac{x\,k\,Q}{(a^2+x^2)^{3/2}}\,\hat{\imath}$$

$$\downarrow$$

$$d\vec{E}_{\text{thin ring}} = \frac{x\,k\,dq}{(a^2+x^2)^{3/2}}\,\hat{\imath}$$

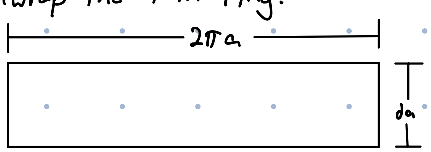
### dq in terms of da

$$\sigma = \frac{Q}{A} = \frac{dq}{dA} \rightarrow dq = \frac{Q}{A}\,dA = \frac{Q}{\pi R^2}\,2\pi a\,da$$

$A = \pi R^2$

$$= \frac{2Qa}{R^2}\,da$$

unwrap the thin ring:




$dA = 2\pi a\,da$