<u>Parallel</u> - When the <u>voltage across</u> circuit elements must be the same due to conductors having the same electric potential.

$$\boxed{\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

$$= \frac{R_1 R_2}{R_2 + R_1}$$

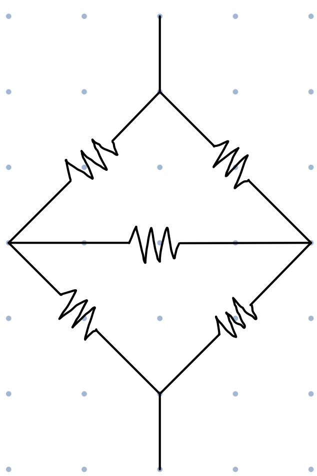
$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 R_2 R_3}} = \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$$

$$= \frac{R_1 R_2 R_3 R_4}{R_2 R_3 R_4 + R_1 R_3 R_4 + R_1 R_2 R_4 + R_1 R_2 R_3}$$

<u>Series</u> - When the <u>current through</u> circuit elements must be the same due to charge being conserved.

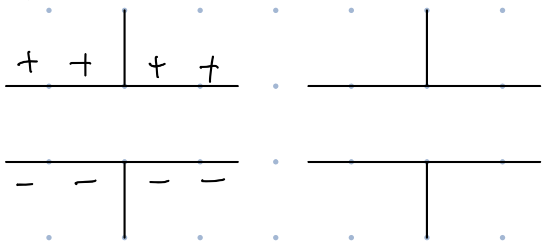
$$\boxed{R_{total} = R_1 + R_2 + R_3 + \dots}$$

Ex of something that is neither parallel nor series: Bridge circuit

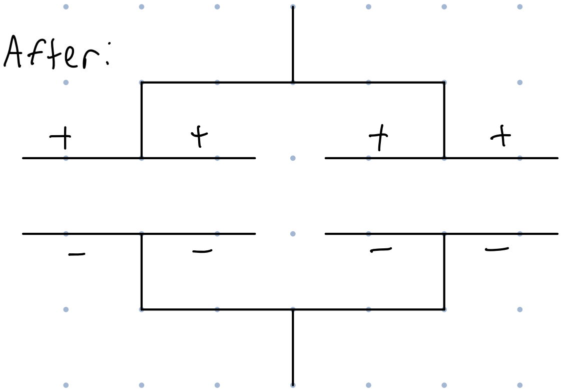


Capacitors in parallel

Before:



After:



$Q_i = Q_f$ (Charge is conserved)

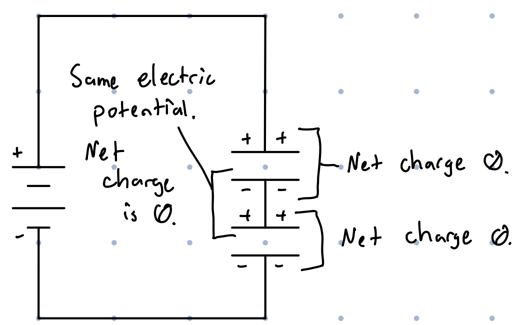
$$\boxed{C_{\parallel} = C_1 + C_2}$$

$\frac{\uparrow A \varepsilon_0}{D} = \uparrow C$ Area increases so capacitance increases.

$\uparrow C = \frac{Q}{\downarrow \Delta V}$ Capacitance increases so voltage decreases.

When charge accelerates it emits light so $V_i > V_f$

Capacitors in series



An ideal battery maintains a voltage across its terminals no matter the current delivered.

- A fully charged capacitor has no current.

Ohm's law

Voltage causes current.

$\boxed{V = IR}$ $\boxed{I = \frac{\Delta Q}{\Delta t} = \frac{dq}{dt}}$ $1 \text{ Amp} = \frac{1 \text{ Coulomb}}{1 \text{ sec}}$

ΔV

Current - How much charge passes a point per unit of time.

Use $C = \frac{Q}{\Delta V} \rightarrow \Delta V = \frac{Q}{C}$ for caps.

- Current doesn't flow. Positive charge flows from high to low potential and that's current.

- Emf - Electric motive force - Voltage generated by a source.

Power

$$P = \frac{\Delta U}{\Delta t} = \frac{dU}{dt} = \frac{dq}{dt} V = IV$$

$$\Delta U = q \Delta V$$

$$dU = dq V$$

$$= I^2 R$$

$$= \frac{V^2}{R}$$

$$V = IR \rightarrow I = \frac{V}{R}$$

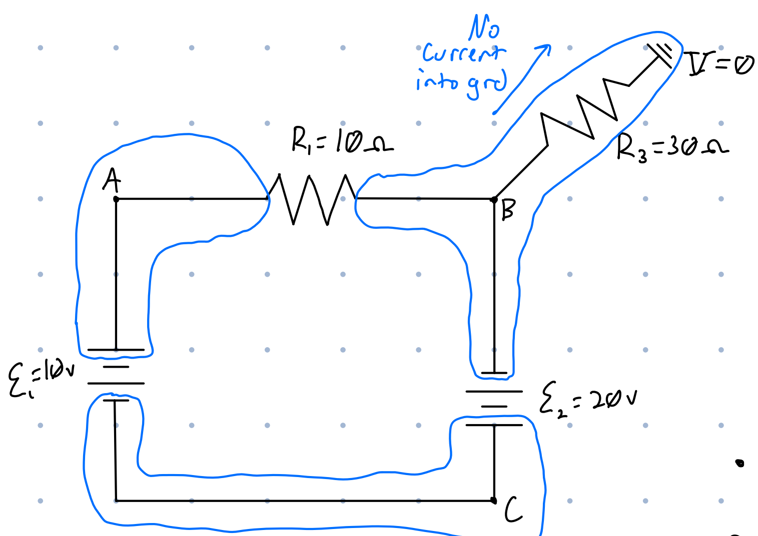
Units: $1 \frac{\text{Joul}}{\text{sec}} = 1$ Watt $\approx$ 20 callories

Ways to solve circuits/Find all the currents.

1) Goofy method
2) Series and parallel ohm's law
3) Kirchhoff's laws

Goofy method

You can only use the goofy method if you're given the electric potential of a surface.



- Current doesn't flow into ground for DC-circuits. It just tells you the electric potential there is 0.

- There are 3 equipotential surfaces (A, B, & C).

- Make your way around the circuit starting with the electric potential surface given.

$V_C = V_B + \varepsilon_2 = 20V$

$V_A = V_C + \varepsilon_1 = 30V$

Ohm's laws for currents

$V_{R_1} = 30 = I_1 R_1$

$I_1 = \frac{30}{10} = 3A$

Series and parallel ohm's law

1^{st} do ohm's law local per elements.

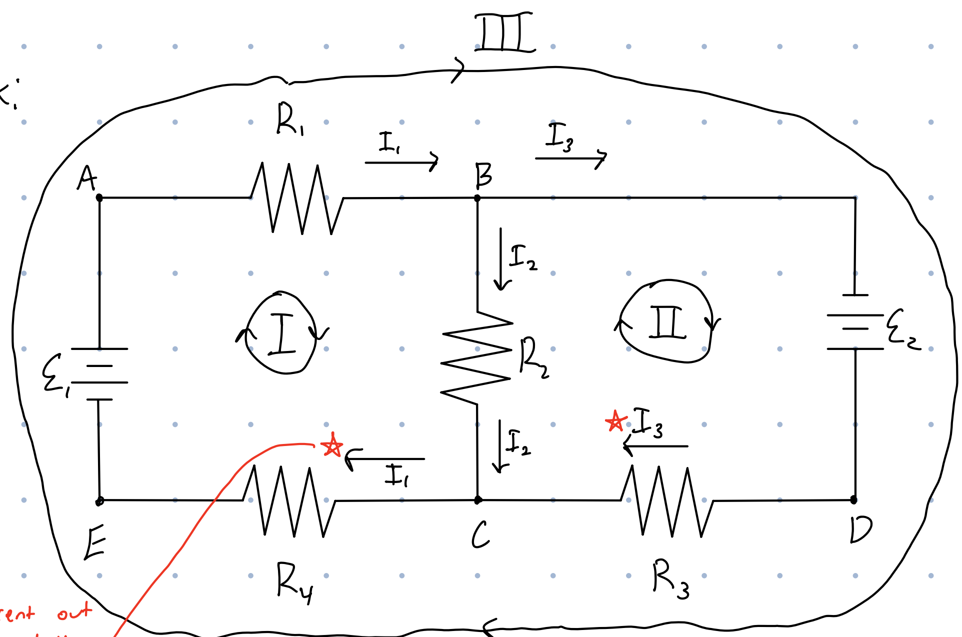
2^{nd} do ohm's law global with total I and R.

Kirchhoff's laws

1) The current into a node equals the current out.
   Conservation of charge.

2) The sum of all voltages in a loop is $\varnothing$.

Ex:



Current out of a battery has to equal current in.

Loops don't need batteries in them.

1) Guess the direction of the currents.

Node B: $I_{in} = I_{out}$

$I_1 = I_2 + I_3$

Node C: $I_{in} = I_{out}$

$I_2 = I_1 + I_3$

2) The direction of the loop doesn't matter.

Loop I:

$\varepsilon_1 - V_{R1} - V_{R2} - V_{R4} = \varnothing$

$\varepsilon_1 - I_1 R_1 - I_2 R_2 - I_1 R_4 = \varnothing$

Loop II:

$\varepsilon_2 - V_{R3} + V_{R2} = \varnothing$

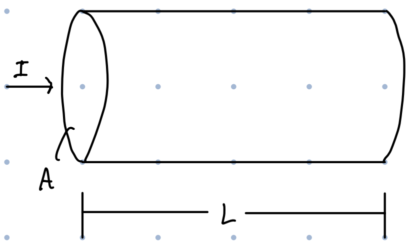
$\varepsilon_2 - I_3 R_3 + I_2 R_2 = \varnothing$

Loop III:

$\varepsilon_1 - V_{R1} + \varepsilon_2 - V_{R3} - V_{R4} = \varnothing$

$\varepsilon_1 - I_1 R_1 + \varepsilon_2 - I_3 R_3 - I_1 R_4 = \varnothing$

Resistivity



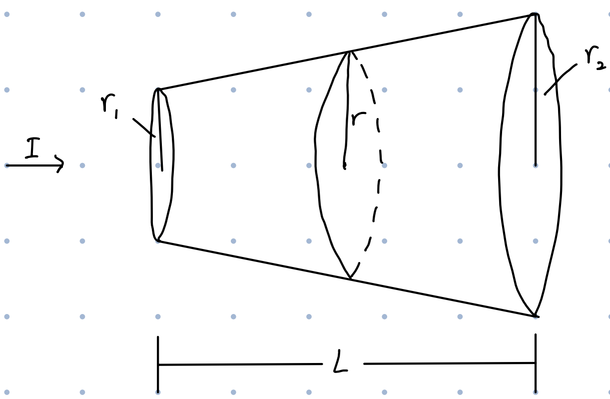
ρ - Resistivity of a substance.

$$\boxed{R = \rho \frac{L}{A}}$$

$\uparrow L \rightarrow \uparrow R$

$\uparrow A \rightarrow \downarrow R$

Ex: Find the resistivity.



$R_{total} = \int dR$ (dR in series)

$= \int \rho \frac{dx}{\pi r^2}$

$= \frac{\rho}{\pi} \int_0^L \frac{1}{\left(\frac{r_2 - r_1}{L} x + r_1\right)^2} dx$

r in terms of x

$r(0) = r_1$

$r(L) = r_2$

Linear

$m = \frac{r_2 - r_1}{L - 0}$

$r - r_1 = m(x - 0)$

$r = \frac{r_2 - r_1}{L} x + r_1$

Ex 2:

Find resistance

$R = \int dR$ (dR in series)

$$= \int_0^R \rho \frac{dy}{\pi r^2} + \int_R^{2R} \rho \frac{dy}{\pi r^2}$$

r in terms of y

Equation of a circle with radius R

$$x^2 + y^2 = R^2$$

Input y. Output: x which is also r

$$r = x = \sqrt{R^2 - y^2}$$

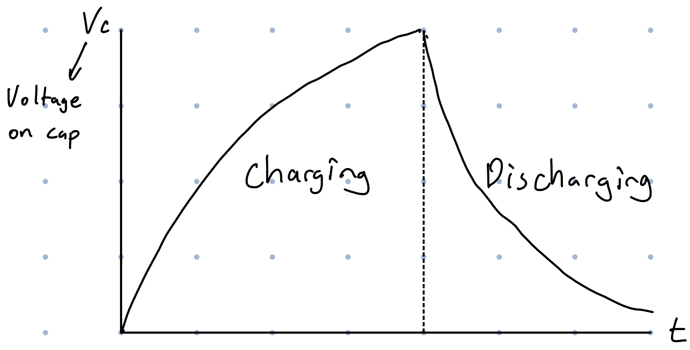
Don't need ± because we just care about the top hemisphere

$$\int_0^R \rho \frac{dy}{\pi r^2} = \int_R^{2R} \rho \frac{dy}{\pi r^2}$$

Resistance from top hemi-sphere = resistance from bottom hemisphere because constant ρ.

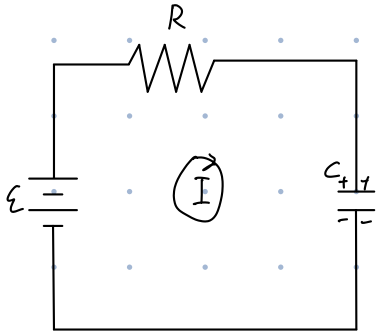
$$= 2 \frac{\rho}{\pi} \int_0^R \frac{1}{R^2 - y^2} \, dy$$

Equations for a capacitor



$$C = \frac{q_c}{V_c} \rightarrow V_c = \frac{q_c}{C}$$

Charging



Loop I:

$$\varepsilon - V_R - V_C = 0$$

$$\varepsilon - \frac{dq}{dt}R - \frac{q}{C} = 0$$

$$\varepsilon - \frac{q}{C} = \frac{dq}{dt}R$$

$$-\varepsilon + \frac{q}{C} = -\frac{dq}{dt}R$$

$$\frac{q - C\varepsilon}{C} = -\frac{dq}{dt}R$$

$$\Big\downarrow \text{Flip}$$

$$\frac{C}{q - C\varepsilon} = -\frac{dt}{R\,dq}$$

$$\frac{dq}{q - C\varepsilon} = -\frac{dt}{RC}$$

$$\int_{Q_i}^{Q_f} \frac{1}{q - C\varepsilon}\,dq = -\frac{1}{RC}\int_0^t dt$$

$$\ln|q - C\varepsilon| \Big|_{Q_i}^{Q_f} = -\frac{t}{RC}$$

$$\ln\left|\frac{Q_f - C\varepsilon}{Q_i - C\varepsilon}\right| = -\frac{t}{RC}$$

$$\frac{Q_f - C\varepsilon}{Q_i - C\varepsilon} = e^{-\frac{t}{RC}}$$

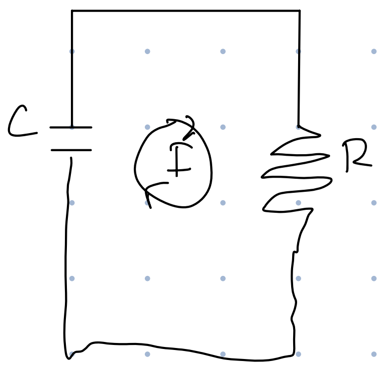
$$Q_f - C\varepsilon = e^{-\frac{t}{RC}}(Q_i - C\varepsilon)$$

$$Q_f = C\varepsilon + (Q_i - C\varepsilon)e^{-\frac{t}{RC}}$$

$$\Big\downarrow \text{If } Q_i = 0$$

$$\boxed{Q_f = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)}$$

<u>Discharging</u>



Loop I:

$$V_C - V_R = 0$$

$$\frac{q_C}{C} = \frac{dq_R}{dt} R$$

$$Q_i = q_C + q_R$$

$$Q_i - q_C = q_R$$

$$\frac{q_C}{C} = \frac{d(Q - q_C)}{dt} R$$

$$\frac{q_C}{C} = \cancel{\frac{dQ}{dt}}^{\,0} - \frac{dq_C}{dt} R \rightarrow -\frac{q_C}{RC} = \frac{dq_C}{dt}$$

$$\frac{C}{q_C} = -\frac{dt}{R\,dq_C}$$

$$\frac{1}{q_C} dq_C = -\frac{1}{CR} dt$$

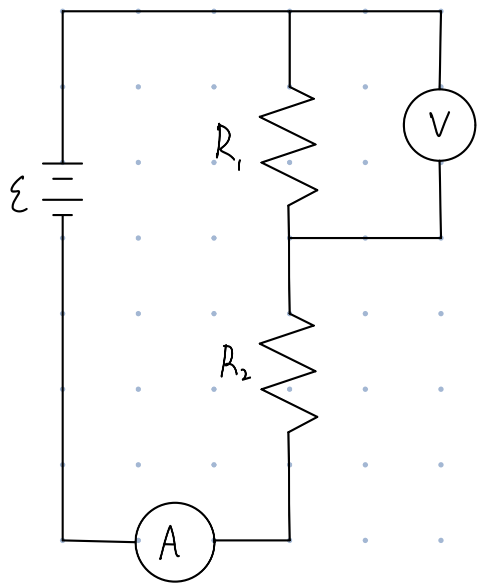
$$\int_{Q_i}^{Q_f} \frac{1}{q_C} dq_C = -\frac{1}{CR} \int_0^{t_f} dt$$

$$\ln\left(\frac{Q_f}{Q_i}\right) = -\frac{t_f}{CR}$$

$$\frac{Q_f}{Q_i} = e^{-\frac{t}{CR}}$$

$$\boxed{Q_f = Q_i e^{-\frac{t}{CR}}}$$

Meters



Want R_A to be $\emptyset$.

Want R_v to be ∞.

$$\varepsilon = I(R_1 + R_2)$$

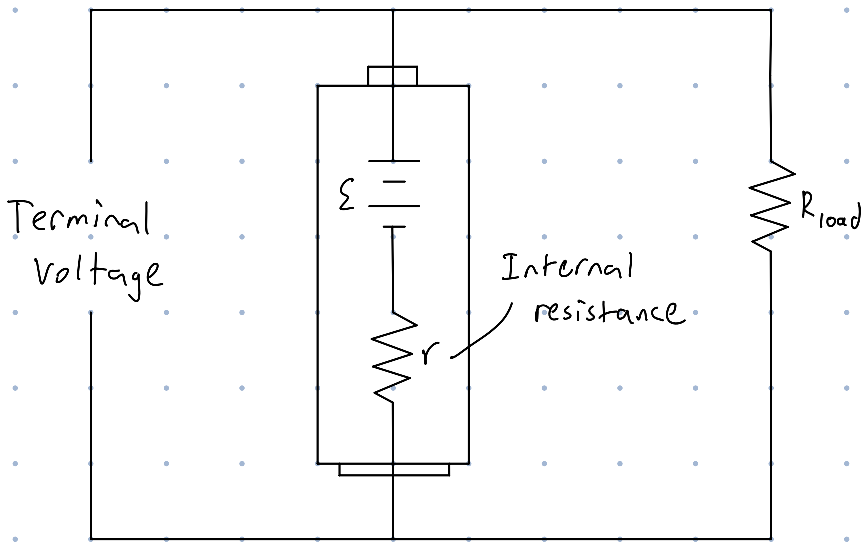
$$I_{\substack{total \\ before}} = \frac{\varepsilon}{(R_1 + R_2)} \qquad V_{\substack{R1 \\ before}} = I_b R_1$$

$$I_{\substack{total \\ after}} = \frac{\varepsilon}{\frac{R_1 r}{R_1 + r} + R_2} \qquad V_{\substack{R1 \\ after}} = I_a \frac{R_1 r}{R_1 + r}$$

V_{R1} before attaching voltmeter?

If you know the R of a meter can you calculate the voltage/current before the meter was in there.

Real batteries



- A dead battery has a high internal resistance(r).
- Terminal voltage drops the more current is delivered.

How to measure r?

Loop I:

$$\mathcal{E} - V_R - V_r = 0$$

$$\mathcal{E} - IR - Ir = 0$$

$$\mathcal{E} = IR + Ir$$

$$\mathcal{E} = I(R + r)$$

$$\frac{\mathcal{E}}{I} = R + r$$

$$r = \frac{\mathcal{E}}{I} - R$$