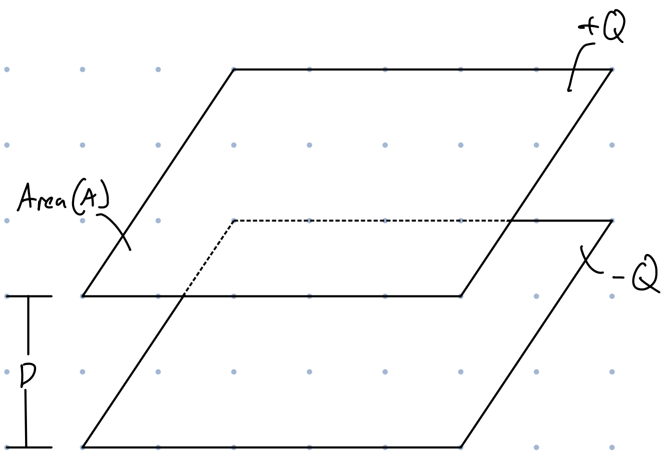
Capacitance

For a single body: $\boxed{C = \frac{Q}{V}}$

Capacitance of a sphere: $C = \frac{Q}{\frac{kQ}{R}} = \frac{R}{k}$

Capacitors



Capacitance of // plate capacitor:

$$\boxed{C = \frac{|Q|}{|\Delta V|}}$$

$|Q|$ ← Charge on one plate

$|\Delta V|$ ← Absolute value of the Voltage across plates

Never negative

In terms of A and D:

$$C = \frac{|Q|}{|\Delta V|}$$

$$\sigma = \frac{Q}{A} \rightarrow Q = \sigma A$$

$$\vec{E}_{inside} = \vec{E}_{+\,plate} + \vec{E}_{-\,plate}$$

Gauss's Law

$$= 2\left(\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$$

$$\Delta V = -\int \vec{E}_{inside} \cdot d\vec{l}$$

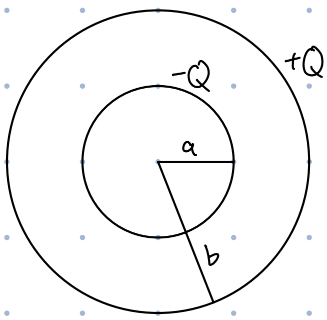
$$= -\frac{\sigma}{\varepsilon_0}\int_0^D dl$$

$$= -\frac{\sigma D}{\varepsilon_0}$$

$$= \frac{\sigma A}{\frac{\sigma D}{\varepsilon_0}}$$

$$C = \frac{A\varepsilon_0}{D}$$

Spherical capacitance



$$C = \frac{Q}{\Delta V}$$

$$C = \frac{Q}{\frac{KQ(a-b)}{ab}}$$

$$C = \frac{ab}{KQ(a-b)}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{l}$$

Guass's law:

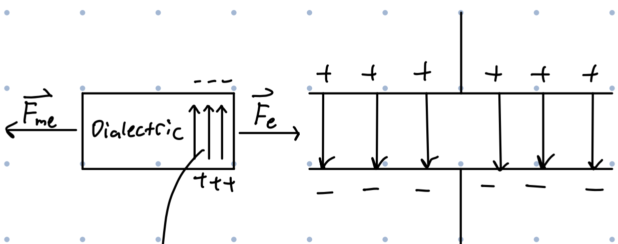
$$E 4\pi r^2 = \frac{Q}{\varepsilon_0} \rightarrow E = \frac{Q}{4\pi r^2 \varepsilon_0} = \frac{KQ}{r^2}$$

$$\Delta V = -\int \frac{KQ}{r^2} dr = -KQ \left[-\frac{1}{r}\right]_a^b$$

$$= KQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$= KQ\left(\frac{a-b}{ab}\right) = \frac{KQ(a-b)}{ab}$$

Dialectric in capacitors:



It has its own E-field by the polarization of charge.

$$\vec{W}_{me} = \int \vec{F}_{me} \cdot d\vec{l}$$

$\vec{F}_{me}$ is opposite to displacement $(d\vec{l})$.

$$= -\int F_{me} \, dl$$

$$= -\Delta V$$

inserting a dialectric decreases the capacitor's energy.

Dialectric constant (K)

$$K = \frac{E_{cap}}{E_{after}}$$

$\leftarrow$ Before you put the dialectric in.

$\leftarrow$ After

$$\vec{E}_{inside} = \vec{E}_{cap} + \vec{E}_{dialectric}$$

$\leftarrow$ Opposite to $\vec{E}_{cap}$

Inserting a dialectric decreases the capacitor's E-field inside.

$$\varepsilon = \varepsilon_0 K$$

Capacitance with dialectric: $C = \frac{A\varepsilon}{D} = \frac{A\varepsilon_0 K}{D}$

$$\frac{A\varepsilon_0 \uparrow K}{D} = \uparrow C$$

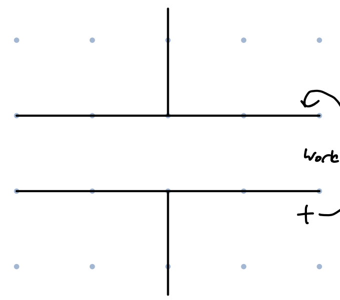
$$\rightarrow \frac{Q^2}{2\uparrow C} = \downarrow V$$

$$\rightarrow \uparrow C = \frac{Q}{\downarrow \Delta V}$$

Inserting a dialectric: $\uparrow C, \downarrow V, \downarrow \Delta V$

U of capacitors

Work done to move charge onto the plates?



Work to move a little + charge across ΔV ↓ Increases as you add more charge.

$$W_{me} = q\Delta V$$

$$dW_{me} = dq\,\Delta V$$

$$W_{me} = \int dW_{me}$$

$$= \int dq\,\Delta V$$

$$C = \frac{q}{\Delta V} \rightarrow \Delta V = \frac{q}{C}$$

$$= \int_0^Q \frac{q}{C}\,dq$$

$$= \frac{Q^2}{2C}$$

Energy density

Find the U of a capacitor in terms of E.

$$U = \frac{Q^2}{2C} \qquad C = \frac{Q}{\Delta V}$$

$$= \frac{Q^2}{2\frac{Q}{\Delta V}} = \frac{Q\Delta V}{2}$$

$$\Delta V_{cap} = -\int_0^D \vec{E}\cdot d\vec{l}$$

E is uniform in the cap ↓ | Just care about magnitude.

$$= Ed$$

$$\vec{E}_{inside} = \vec{E}_{+\,plate} + \vec{E}_{-\,plate}$$

Gauss's Law ↓

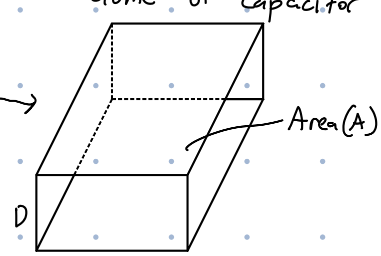
$$= 2\left(\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{\frac{Q}{A}}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

$$Q = EA\varepsilon_0$$

$$= \frac{EA\varepsilon_0(Ed)}{2}$$

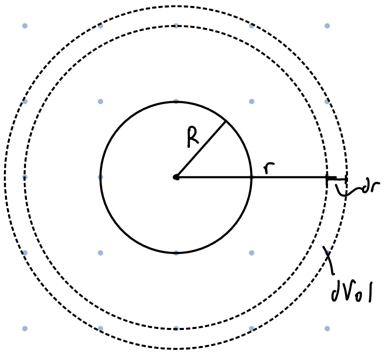
$$= \frac{1}{2}\varepsilon_0 E^2 (Ad)$$



$$\boxed{u_E = \frac{1}{2}\varepsilon_0 E^2} \qquad \boxed{U = \int u_E\, dVol}$$

This works for any E-fields, not just for caps.

How much work to put a sphere of charge Q together?



System: sphere

$$W_{\substack{net \\ ext}} = \Delta \text{Energy}$$

$$@\Delta K = 0 \quad = \cancel{\Delta K}^{\,0} + \Delta U$$

$$= U_f - \cancel{U_i}^{\,0} \text{ (No sphere to begin with)}$$

$dVol = 4\pi r^2 \, dr$

$$= \int u_E \, dVol$$

Since you're outside the sphere, you can treat it as a point charge.

$E = \frac{kQ}{r^2}$

$$= \int \frac{1}{2} \varepsilon_0 E^2 \, 4\pi r^2 \, dr$$

$$= \int \frac{1}{2} \varepsilon_0 \left(\frac{kQ}{r^2}\right)^2 4\pi r^2 \, dr$$

$\varepsilon_0 = \frac{1}{4\pi k}$

$$= \frac{k^2 Q^2 4\pi \varepsilon_0}{2} \int_R^\infty \frac{1}{r^2} \, dr$$

You're adding up the energy from the E-fields that's inside a thin spherical shell from R to ∞.

$$= \frac{kQ^2}{2} \left[-\frac{1}{r}\right]_R^\infty = -\frac{kQ^2}{2}\left(\frac{1}{\infty} - \frac{1}{R}\right)$$

$$= \frac{kQ^2}{2R}$$