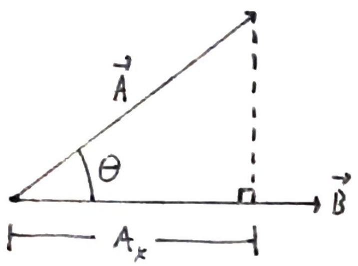
Dot and Cross Product

Dot product:



$$\vec{A} \cdot \vec{B} = A_x B = A\cos(\theta) B = \vec{B} \cdot \vec{A}$$

$$= (A_x \hat{x} + A_y \hat{y}) \cdot (B_x \hat{x} + B_y \hat{y})$$

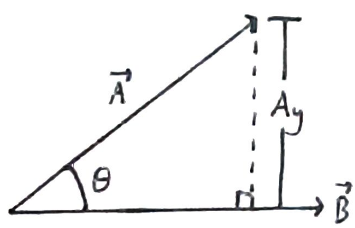
$$= A_x B_x (\hat{x} \cdot \hat{x}) + A_x B_y (\hat{x} \cdot \hat{y}) + A_y B_x (\hat{y} \cdot \hat{x}) + A_y B_y (\hat{y} \cdot \hat{y})$$

$\hat{x} \cdot \hat{x} = (1)(1)\cos(0) = 1 \quad \hat{x} \cdot \hat{y} = (1)(1)\cos(90^\circ) = 0$

$\hat{y} \cdot \hat{y} = (1)(1)\cos(0) = 1 \quad \hat{y} \cdot \hat{x} = (1)(1)\cos(90^\circ) = 0$

$$= A_x B_x + A_y B_y$$

Cross product:



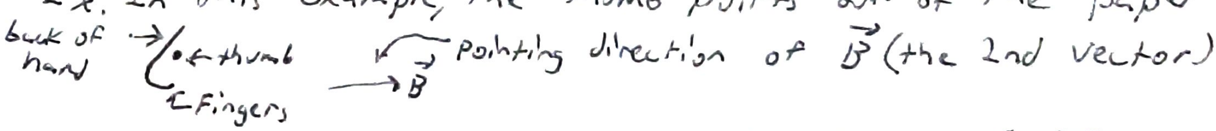
$$\vec{A} \times \vec{B} = A_y B = A\sin(\theta) B$$

The cross product also has a direction, so $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

The direction is determined by the right hand rule.

Fingers the same direction as the 1st vector, have them curle in the pointing direction of the 2nd vector, and the direction the thumb points to is the cross product's direction.

Ex: In this example, the thumb points out of the paper



$$\vec{A} \times \vec{B} = \text{Determinant of} \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} = \hat{x} \begin{bmatrix} A_y & A_z \\ B_y & B_z \end{bmatrix} - \hat{y} \begin{bmatrix} A_x & A_z \\ B_x & B_z \end{bmatrix} + \hat{z} \begin{bmatrix} A_x & A_y \\ B_x & B_y \end{bmatrix}$$

$$\text{Determinant of} \begin{bmatrix} A_x & A_y \\ B_x & B_y \end{bmatrix} = A_x B_y - A_y B_x$$

$$= \hat{x}(A_y B_z - A_z B_y) - \hat{y}(A_x B_z - A_z B_x) + \hat{z}(A_x B_y - A_y B_x)$$