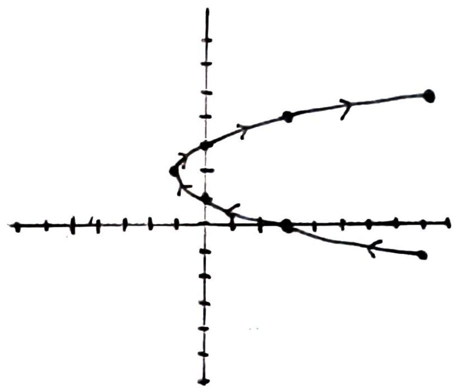
Parametric Equations

Parametric equations express X and Y in terms of a 3^{rd} variable.

Ex: Graph $x = t^2 - 2t$ and $y = t+1$

t	x	y
-2	8	-1
-1	3	Ø
Ø	Ø	1
1	-1	2
2	Ø	3
3	3	4
4	8	5



Sketch the curve with increasing values of t.

Finding a Cartesian parameter:

1. Use one of the equations to solve for t
2. Plug in t into the other equation

$y = t+1 \Rightarrow t = y-1$

$x = t^2 - 2t \Rightarrow x = (y-1)^2 - 2(y-1) \Rightarrow x = y^2 - 4y + 3$

Finding derivatives:

chain rule: $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$$\boxed{\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}} \qquad \frac{d}{dx}\left(\frac{dy}{dx}\right) = \boxed{\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}}$$

$$\frac{dy}{dt} = 1 \qquad \frac{dx}{dt} = 2t - 2 \qquad \frac{d}{dt}\left(\frac{dy}{dx}\right) = -\frac{1}{(2t-2)^2}(2)$$

$$\frac{dy}{dx} = \frac{1}{2t-2} \qquad \frac{d^2y}{dx^2} = \frac{-\frac{2}{(2t-2)^2}}{2t-2} = -\frac{2}{(2t-2)^3}$$