Finding Integrals Overview

1. Simplify using algebra or trig identities.

Ex:
$$\int\sqrt{\frac{1-x}{1+x}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{1+x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}\,dx = \int \frac{1-x}{\sqrt{1-x^2}}\,dx$$

$$= \int \frac{1}{\sqrt{1-x^2}}\,dx - \int\frac{x}{\sqrt{1-x^2}}\,dx \qquad \begin{array}{l} u = 1-x^2 \\ du = -2x\,dx \Rightarrow dx = \dfrac{du}{-2x}\end{array}$$

$$= \sin^{-1}x - \int \frac{x}{\sqrt{u}}\,\frac{du}{-2x} = \sin^{-1}x + \frac{1}{2}\int\frac{1}{\sqrt{u}}\,du = \sin^{-1}x + \frac{1}{2}\left[2\sqrt{u}\right] + C$$

$$= \sin^{-1}x + \sqrt{1-x^2} + C$$

2. Look for an easy u-sub
3. Categorize

<u>Powers of trig</u>: sin and cos, tan and sec, and cot and csc. Sin A/cos B

Ex: $\int \sec^2 x \tan^3 x\,dx$

<u>Radicals</u>: Trig inverse substitution Ex: $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

<u>Rational functions</u>: Method of partial fractions Ex: $\int \frac{3x^2+4x-2}{x^3+2x}\,dx$

<u>Multiplication of different types of functions</u>: Integration by parts.

Ex: $\int e^{-2x}\cos x\,dx$

4. Other

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad\qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx \qquad u = \cos x$$

$$du = -\sin x \, dx \Rightarrow dx = \frac{du}{-\sin x}$$

$$= \int \frac{\sin x}{u} \cdot \frac{du}{-\sin x} = -\int \frac{1}{u} \, du = -\ln|u| + C = -\ln|\cos x| + C$$

$$= \ln 1 - \ln|\cos x| + C = \ln\left|\frac{1}{\cos x}\right| + C = \ln|\sec x| + C$$

$$\int \sec x \, dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \, dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

$$u = \sec x + \tan x$$

$$du = \sec x \tan x + \sec^2 x \, dx$$

$$= \int \frac{1}{u} \, du = \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$

$$\int \frac{x^2}{x^2 - a^2} \, dx \text{ or } \int \frac{x^2}{x^2 + a^2} \, dx = \int \frac{x^2 + a^2 - a^2}{x^2 + a^2} \, dx$$

$$= \int \frac{x^2 + a^2}{x^2 + a^2} - \frac{a^2}{x^2 + a^2} \, dx = \int dx - a \int \frac{a}{x^2 + a^2} \, dx$$

$$= x - a \tan^{-1}\left(\frac{x}{a}\right) + C$$
