Continuity

Continious at a point:

1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ is defined
3. $\lim_{x \to a} f(x) = f(a)$

Types of discontinuities:

- Removed
- Jump
- Infinite

- Continious at the enpoint means it's continious from only the left or right.
- If $f(a)$ is differentiable, then $f(a)$ is also continious.

Finding domains:

- These are also continious if $f(x)$ and $g(x)$ are also.

  $f+g$, $f-g$, $f \cdot g$, $\frac{f}{g}$ if $g(a) \neq 0$

  - The domain is the overlapping domains of f and g.

- $f(g(x)) = f \circ g$
  1. Find domain of $g(x)$
  2. Find domain of $f(x)$
  3. Map domain of $f(x)$ to domain of $g(x)$
  4. Find the overlap of the domain of $g(x)$ and the mapped domain

  Ex: Find domain of $\sqrt{1+\frac{5}{x}}$ $\quad f(x) = \sqrt{g(x)}$ $\quad g(x) = 1 + \frac{5}{x}$

  Domain of $g(x)$: $x \neq 0$

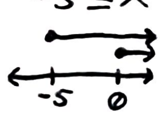
  Domain of $f(x)$: $g(x) \geq 0$

  Map: $1 + \frac{5}{x} \geq 0$ $\quad \frac{5}{x} \geq -1$

  If $x \geq 0$

  $5 \geq -x$

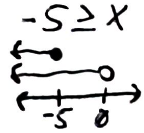
  $-5 \leq x$

  If $x < 0$ ← when multiplying or dividing by a negative num, you need to change the inequality.

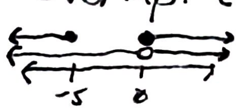
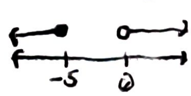
  $5 \leq -x$

  $-5 \geq x$

  $x \geq 0$ or $x \leq -5$

  Overlap: ($x \geq 0$ or $x \leq -5$) and $x \neq 0$

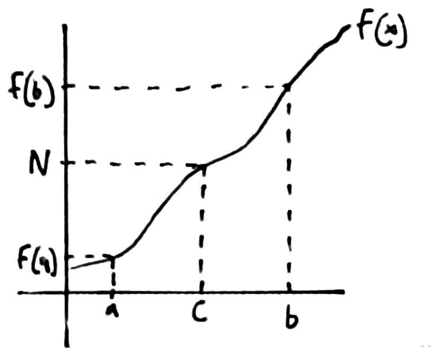
   →  → $(-\infty, -5] \cup (0, \infty)$

- Domains of common functions

polynomial	$(-\infty, \infty)$
$\sqrt{x}$	$[0, \infty)$
$\frac{1}{x}$	$(-\infty, 0) \cup (0, \infty)$
a^x	$(-\infty, \infty)$
$\log_b x$	$(0, \infty)$

sin/cos	$(-\infty, \infty)$
tan	$x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$, etc
$\sin^{-1}/\cos^{-1}$	$[-1, 1]$
$\tan^{-1}$	$(-\infty, \infty)$

Intermediate Value Theorem (IVT):



If $f(x)$ is continious between $[a, b]$, and $f(a) < N < f(b)$, then there exists c between (a, b) such that $f(c) = N$.

Ex: Prove that there's a root of $\cos x + e^x + x = 0$ on the interval $(-1, 0)$

$f(x) = \cos x + e^x + x$

$f(x)$'s domain is $(-\infty, \infty)$, therefore its continious between $[-1, 0]$

$a = -1 \quad b = 0 \quad N = 0$

$f(a) = -0.09 \quad f(b) = 2 \qquad f(-1) < N < f(0)$

$\therefore$ there exists c between $(-1, 0)$ such that $f(c) = N = 0$.